01. (a) α [illegible] β [illegible] $x^2 + px + 1 = 0$ [illegible]

$(x-\alpha)(x-\beta) = x^2 + px + 1 = 0$

$\Rightarrow (\alpha - x)(\beta - x) = x^2 + px + 1 = 0$

[illegible] $(\alpha-\gamma)(\beta-\gamma) = \gamma^2 + p\gamma + 1$ [illegible]

$(\alpha-\delta)(\beta-\delta) = \delta^2 + p\delta + 1$

$\Rightarrow (\alpha-\gamma)(\beta-\gamma)(\alpha-\delta)(\beta-\delta)$

$= (\gamma^2 + p\gamma + 1)(\delta^2 + p\delta + 1)$

[illegible] $\alpha+\beta = -p,\ \alpha\beta = 1,$

$\gamma+\delta = -1/p,\ \gamma\delta = 1$

$(\alpha-\gamma)(\beta-\gamma)(\alpha-\delta)(\beta-\delta)$

$= (\gamma^2 + p\gamma + 1)(\delta^2 + p\delta + 1)$

$= \gamma^2\delta^2 + p\gamma^2\delta + \gamma^2 + p\gamma\delta^2$

$+ p^2\gamma\delta + p\gamma + \delta^2 + p\delta + 1$

$= (\gamma\delta)^2 + p\gamma\delta(\gamma+\delta) + p^2\gamma\delta$

$+ p(\gamma+\delta) + \gamma^2 + \delta^2 + 1$

$= (\gamma\delta)^2 + p(\gamma\delta)(\gamma+\delta) + p^2\gamma\delta$

$+ p(\gamma+\delta) + (\gamma+\delta)^2 - 2\gamma\delta + 1$

$= 1 + p.1\left(\frac{-1}{p}\right) + p^2(1)$

$+ p.\left(\frac{-1}{p}\right) + \left(\frac{-1}{p}\right)^2 - 2.1 + 1$

$= 1 - 1 + p^2 - 1 + \frac{1}{p^2} - 1$

$= p^2 - 2 + \frac{1}{p^2}$

$= \left(p - \frac{1}{p}\right)^2$

(b) $\log_a b = x$ [illegible] $\log_b a = y$ [illegible]

[illegible] $b = a^x \Rightarrow a = b^y$ [illegible]

[illegible] $b = \left[b^y\right]^x$

$b = b^{xy}$

[illegible] $xy = 1$ [illegible]

[illegible] $y = \frac{1}{x} \Rightarrow \log_a b = \frac{1}{\log_b a}$

$\frac{1}{\log_2 2001} + \frac{1}{\log_3 2001} + \frac{1}{\log_4 2001}$

$+ \dots\dots + \frac{1}{\log_{100} 2001}$

$= \log_{2001} 2 + \log_{2001} 3$

$+ \dots\dots \log_{2001} 100$

([illegible])

$= \log_{2001} (2.3.4 \dots\dots 100)$

$= \log_{2001} 100!$

$= \dfrac{1}{\log_{100!} 2001}$

([illegible])

02. (a) $n = 1$ [illegible] $A_n = 1 - (1-\beta)\alpha^{n-1}$ [illegible]

$A_1 = 1 - (1-\beta)\alpha^{1-1} = 1 - (1-\beta) = \beta$

$\therefore n = 1$ [illegible]

$n = p$ [illegible]

[illegible] $A_p = 1 - (1-\beta)\alpha^{p-1}$ [illegible]

$A_{p+1} = (1-\alpha)(1 - A_p) + A_p$

$= (1-\alpha)(1-\beta)\alpha^{p-1}$

$+ (1 - (1-\beta)\alpha^{p-1})$

$= (1-\beta)\alpha^{p-1}(1 - \alpha - 1) + 1$

$= 1 - (1-\beta)\alpha^p$

$= 1 - (1-\beta)\alpha^{\overline{p+1}-1}$

$\therefore n = p$ [illegible] $n = p + 1$ [illegible]

[illegible] n [illegible]

$A_n = 1 - (1-\beta)\alpha^{n-1}$ [illegible]

$\sum_{r=1}^{n} A_r = \sum_{r=1}^{n} (1 - (1-\beta)\alpha^{r-1})$

$= \sum_{r=1}^{n} 1 - (1-\beta)\sum_{r=1}^{n} \alpha^{r-1}$

$= n - (1-\beta)\frac{(1-\alpha^n)}{(1-\alpha)}$

(b) $k\,{}^nC_k = k\,\frac{n!}{k!(n-k)!}$

$= \frac{n(n-1)!}{(k-1)!(n-k)!}$

$= \frac{n(n-1)!}{(k-1)!((n-1)-(k-1))!}$

$= n\,{}^{n-1}C_{k-1}$

$\sum_{k=0}^{n} k\,{}^nC_k\,x^k(1-x)^{n-k}$

$= \sum_{k=1}^{n} k\,{}^nC_k\,x^k(1-x)^{n-k}$

$= \sum_{k=1}^{n} n\,{}^{n-1}C_{k-1}\,x^k(1-x)^{n-k}$

$= nx\sum_{k=1}^{n} {}^{n-1}C_{k-1}\,x^{k-1}(1-x)^{(n-1)-(k-1)}$

$= nx\sum_{m=0}^{n-1} {}^{n-1}C_m\,x^m(1-x)^{(n-1)-m}$

[illegible] $m = k - 1$

$= nx\,[x + (1-x)]^{n-1}$ ([illegible])

$= \underline{\underline{nx}}$

03. (a) G, B, G, B, G, B, ... G, B [illegible]

[illegible] 7!

[illegible] 7!

[illegible] $= 7! \times 7!$

(b) $y = 2|x+1| - 3, \quad x \in \mathbb{R}$

$= \begin{cases} 2(x+1) - 3 & x+1 \geq 0 \\ -2(x+1) - 3 & x+1 < 0 \end{cases}$

$= \begin{cases} 2x - 1 & x \geq -1 \\ -2x - 5 & x < -1 \end{cases}$

$y = x + 2|x-1|,\ x \in \mathbb{R}$

$= \begin{cases} x + 2(x-1) & x - 1 \geq 0 \\ x - 2(x-1) & x - 1 < 0 \end{cases}$

$= \begin{cases} 3x - 2 & x \geq 1 \\ -x + 2 & x < 1 \end{cases}$

[illegible] x [illegible] $(-7, 1) \cup (1, \infty)$

$y = x + 2|x-1| \Rightarrow y = 2|x+1| - 3$

[illegible]

$x < -1$ [illegible]

$x + 2|x-1| = 2|x+1| - 3$

$x - 2(x-1) = -2(x+1) - 3$

$x = -7$

$-1 \leq x < 1$ [illegible]

$x - 2(x-1) = 2(x+1) - 3 \Rightarrow x = 1$

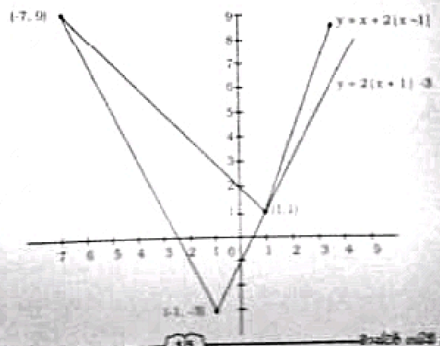
[illegible]

$1 \leq x$ [illegible] $3x - 2 = 2x - 1 \Rightarrow x = 1$

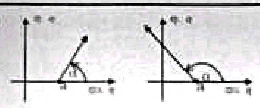
$\therefore x + 2|x-1| = 2|x+1| - 3$

[illegible]

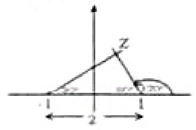
$x = -7$ [illegible] $x = 1$



04. (a)



[illegible]



$Z = 1 - 2\cos 60^\circ \cdot \cos 60^\circ + i\,2\cos 60^\circ \sin 60^\circ$

$= 1 - 2 \times \frac{1}{2} \times \frac{1}{2} + i\,2 \times \frac{1}{2} \times \frac{\sqrt{3}}{2}$

$= \frac{1}{2} + \frac{\sqrt{3}}{2}\,i$

(b) $\frac{3 - i}{2 - 3i} = \frac{(3 - i)(2 + 3i)}{(2 - 3i)(2 + 3i)}$

$= \frac{6 + 9i - 2i - 3i^2}{13} = \frac{13 + 13i}{13}$

$= 1 + i \qquad = \lambda(1 + i)$ [illegible] $\lambda = 1$

$\left(\frac{3 - i}{2 - 3i}\right)^6 = (1 + i)^6$

$= \left[(1 + i)^2\right]^3$

$= \left[1 + 2i + i^2\right]^3$

$= (2i)^3 \quad (\because i^2 = -1)$

$= 8i^3 = -8i$ [illegible]

05. (a) $x = t - \sin t \qquad y = 1 - \cos t$

$\frac{dx}{dt} = 1 - \cos t,\ \frac{dy}{dt} = \sin t$

$\frac{dy}{dx} = \frac{dy}{dt} \cdot \frac{dt}{dx} = \sin t \cdot \frac{1}{1 - \cos t}$

$= \frac{2\sin t/2 \cos t/2}{2\sin^2 t/2}$

$= \cot t/2 \quad (\because \sin t/2 \neq 0)$

$\frac{d^2y}{dx^2} = \frac{d}{dx}\left(\frac{dy}{dx}\right)$

$= \frac{d}{dt}\left(\frac{dy}{dx}\right) \cdot \frac{dt}{dx}$

$= -\frac{1}{2}\operatorname{cosec}^2 t/2 \times \frac{1}{1 - \cos t}$

$= -\frac{1}{2}\operatorname{cosec}^2 t/2 \times \frac{1}{2\sin^2 t/2}$

$= -\frac{1}{4}\operatorname{cosec}^4 t/2$

$\frac{d^3y}{dx^3} = \frac{d}{dt}\left(\frac{d^2y}{dx^2}\right) \cdot \frac{dt}{dx}$

$= -\frac{1}{4} \times 4\operatorname{cosec}^3 t/2 \left(-\operatorname{cosec} t/2 \cot t/2 \cdot \frac{1}{2}\right) \frac{1}{1 - \cos t}$

$= \frac{1}{2}\operatorname{cosec}^4 t/2 \cot t/2 \cdot \frac{1}{2\sin^2 t/2}$

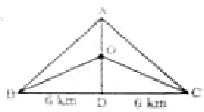
$= \left[-\frac{1}{4}\operatorname{cosec}^4 t/2\right] \cot t/2 \cdot \frac{1}{\sin^2 t/2}$

$\frac{d^3y}{dx^3} = -\left(\frac{d^2y}{dx^2}\right)\left(\frac{dy}{dx}\right) \cdot \frac{2}{1 - \cos t}$

$\frac{d^3y}{dx^3} = -\left(\frac{d^2y}{dx^2}\right)\left(\frac{dy}{dx}\right)\frac{2}{y}$

$\Rightarrow y\frac{d^3y}{dx^3} + 2\left(\frac{d^2y}{dx^2}\right)\left(\frac{dy}{dx}\right) = 0$

(b)



$OD = x$ [illegible]

[illegible] $OA = 16 - x \Rightarrow OC^2 = x^2 + 6^2$

[illegible] l [illegible]

[illegible] $l = (16 - x) + 2\sqrt{x^2 + 36}$

x [illegible]

$\frac{dl}{dx} = -1 + \frac{2}{2}(x^2 + 36)^{-1/2} \times 2x$

$= -1 + \frac{2x}{\sqrt{x^2 + 36}}$

$= \frac{2x - \sqrt{x^2 + 36}}{\sqrt{x^2 + 36}}$

$= \frac{4x^2 - (x^2 + 36)}{\sqrt{x^2 + 36}\,(2x + \sqrt{x^2 + 36})}$

$= \frac{3x^2 - 36}{\sqrt{x^2 + 36}\,(2x + \sqrt{x^2 + 36})}$

$= \frac{3(x - 2\sqrt{3})(x + 2\sqrt{3})}{\sqrt{x^2 + 36}\,(2x + \sqrt{x^2 + 36})}$

$\frac{dl}{dx} = 0,\ (x - 2\sqrt{3})(x + 2\sqrt{3}) = 0$ [illegible]

[illegible] $x = 2\sqrt{3}$ [illegible] $(\because x \neq -2\sqrt{3})$

$\frac{dl}{dx} < 0,\ 0 < x < 2\sqrt{3}$ [illegible]

$\frac{dl}{dx} = 0,\ x = 2\sqrt{3}$ [illegible]

$\frac{dl}{dx} > 0,\ 2\sqrt{3} < x < 16$

[illegible] $0 < x < 2\sqrt{3}$ [illegible] l [illegible]

$2\sqrt{3} < x < 16$ [illegible] l [illegible]

$\therefore x = 2\sqrt{3}$ [illegible] l [illegible]

[illegible]

[illegible] A [illegible] $(16 - 2\sqrt{3})$ km [illegible]

06. (a) $I = \int_1^{\sqrt{2}} \frac{1}{x^2\sqrt{4 - x^2}}\,dx \Rightarrow x = 2\sin\theta$

[illegible] $dx = 2\cos\theta\, d\theta$

$x = 1 \Rightarrow \sin\theta = \frac{1}{2},\ \theta = \pi/6$

$x = \sqrt{2} \Rightarrow \sin\theta = \frac{1}{\sqrt{2}},\ \theta = \frac{\pi}{4}$

[illegible] $I = \int_{\pi/6}^{\pi/4} \frac{2\cos\theta\, d\theta}{4\sin^2\theta\sqrt{4 - 4\sin^2\theta}}$

$= \int_{\pi/6}^{\pi/4} \frac{\cos\theta\, d\theta}{4\sin^2\theta\cos\theta}$

$= \frac{1}{4}\int_{\pi/6}^{\pi/4} \operatorname{cosec}^2\theta\, d\theta$

$= -\frac{1}{4}\left[\cot\theta\right]_{\pi/6}^{\pi/4}$

$= -\frac{1}{4}\left[\cot \pi/4 - \cot \pi/6\right]$

$= -\frac{1}{4}(1 - \sqrt{3}) = \frac{1}{4}(\sqrt{3} - 1)$

(b) $\int_2^4 x \ln x\, dx$

$= \int_2^4 \ln x \frac{d}{dx}(x^2/2)\, dx$

$= \left[(\ln x)\frac{x^2}{2}\right]_2^4 - \int_2^4 \frac{x^2}{2} \cdot \frac{1}{x}\, dx$

$= \left[\frac{x^2}{2}\ln x\right]_2^4 - \frac{1}{2}\left[\frac{x^2}{2}\right]_2^4$

$= \frac{16}{2}\ln 4 - \frac{4}{2}\ln 2 - \frac{1}{2}\left(\frac{16}{2} - \frac{4}{2}\right)$

$= 8\ln 4 - 2\ln 2 - \frac{1}{2} \times \frac{12}{2}$

$= 2(\ln 4^4 - \ln 2) - 3$

$= 2\ln\frac{256}{2} - 3$

$= 2\ln 128 - 3$

(c) $\frac{7x - x^2}{(2 - x)(x^2 + 1)} = \frac{A}{2 - x} + \frac{Bx + C}{x^2 + 1}$ [illegible]

[illegible]

$7x - x^2 = A(x^2 + 1) + (Bx + C)(2 - x)$

$x = 2 \Rightarrow 14 - 4 = 5A \Rightarrow A = 2$

x^2 [illegible] $-1 = A - B \Rightarrow B = 3$

[illegible] $0 = A + 2C \Rightarrow C = -1$

$\int_0^1 \frac{7x - x^2\, dx}{(2 - x)(x^2 + 1)}$

$= \int_0^1 \frac{2}{2 - x}\, dx + \int_0^1 \frac{3x - 1}{x^2 + 1}\, dx$

$= -2\left[\ln|2 - x|\right]_0^1$

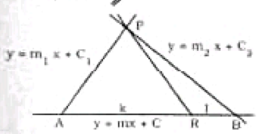
$+ \frac{3}{2}\int_0^1 \frac{2x}{x^2 + 1}\, dx - \int_0^1 \frac{dx}{x^2 + 1}$

$$= -2\left[\ln 1 - \ln 2\right] + \frac{3}{2}\left[\ln(x^2+1)\right]_0^1 - \left[\tan^{-1} x\right]_0^1$$

$$= 2\ln 2 + \frac{3}{2}\left[\ln 2 - \ln 1\right] - \tan^{-1} 1 - 0$$

$$= \frac{7}{2}\ln 2 - \pi/4$$

(ආ)



$A = (x_1, y_1)$ හා $B = (x_2, y_2)$ [illegible]

A හා B $y = mx + C$ [illegible]

$$\left.\begin{array}{l} y_1 = mx_1 + C \\ y_2 = mx_2 + C \end{array}\right\} \Rightarrow y_1 - y_2 = m(x_1 - x_2)$$

A, $y = m_1 x + C_1$ [illegible]

$y_1 = m_1 x_1 + C_1 \Rightarrow y_1 - m_1 x_1 - C_1 = 0$ —— ❶

B, $y = m_2 x + C_2$ [illegible]

$y_2 = m_2 x_2 + C_2 \Rightarrow y_2 - m_2 x_2 - C_2 = 0$ —— ❷

[illegible] $R = \left[\frac{kx_2 + x_1}{k+1}, \frac{ky_2 + y_1}{k+1}\right]$ [illegible]

PR [illegible] $u_1 + \lambda u_2 = 0$ [illegible]

[illegible] $(y - m_1 x - C_1) + \lambda(y - m_2 x - C_2) = 0$

[illegible] R [illegible]

$$\left[\frac{ky_2 + y_1}{k+1} - m_1\left(\frac{kx_2 + x_1}{k+1}\right) - C_1\right]$$

$$+\lambda\left[\frac{ky_2 + y_1}{k+1} - m_2\left(\frac{kx_2 + x_1}{k+1}\right) - C_2\right] = 0$$

$\Rightarrow (ky_2 + y_1) - m_1(kx_2 + x_1) - C_1(k+1)$

$+ \lambda\left[(ky_2 + y_1) - m_2(kx_2 + x_1) - C_2(k+1)\right] = 0$

$(y_1 - m_1 x_1 - C_1) + k(y_2 - m_1 x_2 - C_1)$

$+ \lambda\left[(y_1 - m_2 x_1 - C_2) + k(y_2 - m_2 x_2 - C_2)\right] = 0$

$k(y_2 - m_1 x_2 - C_1) + \lambda(y_1 - m_2 x_1 - C_2) = 0$

[illegible] ❶ හා ❷ [illegible]

$\Rightarrow k\left[y_2 - m_1 x_2 - (y_1 - m_1 x_1)\right]$

$+ \lambda\left[y_1 - m_2 x_1 - (y_2 - m_2 x_2)\right] = 0$

$\Rightarrow k\left[y_2 - y_1 - m_1(x_2 - x_1)\right]$

$+ \lambda\left[y_1 - y_2 - m_2(x_1 - x_2)\right] = 0$

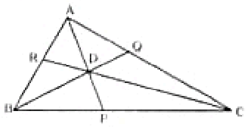
$$\Rightarrow k\left[-\frac{y_1 - y_2}{x_1 - x_2} + m_1\right] + \lambda\left[\frac{y_1 - y_2}{x_1 - x_2} - m_2\right] = 0$$

$\Rightarrow k(-m + m_1) + \lambda(m - m_2) = 0$

$$\lambda = k\frac{(m - m_1)}{(m - m_2)}$$

∴ PR [illegible]

$$u_1 + k\left(\frac{m - m_1}{m - m_2}\right)u_2 = 0$$



$u_{AB} = y + \frac{3}{2}x - 3 = 0$ —— ❶

$u_{BC} = y + 2x - 2 = 0$ —— ❷

හා $u_{AC} = y + x - 3 = 0$ —— ❸ [illegible]

$m_{AB} = -3/2,\ m_{BC} = -2,\ m_{AC} = -1$

(i) ❶ හා ❸ [illegible] $\frac{1}{2}x = 0 \Rightarrow x = 0$

[illegible] ❸ [illegible] $y = 3$

$A = (0, 3)$

(ii) CR [illegible]

$$u_{CA} + k\left(\frac{m_{AB} - m_{CA}}{m_{AB} - m_{BC}}\right)u_{BC} = 0$$ [illegible]

[illegible] $k = \frac{1}{2}$, $AR = \frac{1}{2}RB$ [illegible]

[illegible]

$$x + y - 3 + \frac{1}{2}\left[\frac{-\frac{3}{2} + 1}{-\frac{3}{2} + 2}\right](2x + y - 2) = 0$$

$$\Rightarrow x + y - 3 + \frac{1}{2}(-1)(2x + y - 2) = 0$$

$$\cancel{x} + y - 3 - \cancel{x} - y/2 + 1 = 0$$

$\Rightarrow y - 4 = 0$ [illegible] CR [illegible]

BQ [illegible]

$$u_{AB} + k\left(\frac{m_{AC} - m_{AB}}{m_{CA} - m_{BC}}\right)u_{BC} = 0$$

[illegible] $k = 2/3$ (∵ $AQ = \frac{2}{3}QC$)

[illegible]

$$\frac{3}{2}x + y - 3 + \frac{2}{3}\left[\frac{-1 + 3/2}{-1 + 2}\right](2x + y - 2) = 0$$

[illegible]

$$\frac{3}{2}x + y - 3 + \frac{\cancel{2}}{3}\left[\frac{1}{\cancel{2}}\right](2x + y - 2) = 0$$

$$9x + 6y - 18 + 2(2x + y - 2) = 0$$

$$13x + 8y - 22 = 0$$

(iii) ❶ හා ❷ [illegible]

$x = -2 \Rightarrow y = 6$ ∴ $B = (-2, 6)$

D [illegible]

$u_{BQ} + \lambda u_{CR} = 0$ [illegible]

[illegible] $13x + 8y - 22 + \lambda(y - 4) = 0$ [illegible]

[illegible] AD [illegible] $A = (0, 3)$ [illegible]

$0 + 24 - 22 + \lambda(3 - 4) = 0 \Rightarrow \lambda = 2$

∴ AD [illegible]

$13x + 10y - 30 = 0$ — ❹

AD හා BC [illegible] P [illegible]

❷ හා ❹ [illegible]

$13x + 10(2 - 2x) - 30 = 0$

$-7x - 10 = 0 \Rightarrow x = \frac{-10}{7}$

[illegible] $y = 2 - 2\left(\frac{-10}{7}\right) = \frac{34}{7}$

$\therefore P = \left(\frac{-10}{7}, \frac{34}{7}\right)$

[illegible] $A = (0, 3)$, $B = (-2, 6)$ හා

$$P = \left(\frac{-10}{7}, \frac{34}{7}\right)$$

$$AP^2 = \frac{100}{49} + \frac{169}{49} = \frac{269}{49} \Rightarrow AP = \frac{\sqrt{269}}{7}$$

$$PB^2 = \frac{16}{49} + \frac{64}{49} = \frac{80}{49} \Rightarrow PB = \frac{\sqrt{80}}{7}$$

$AP : PB = \sqrt{269} : \sqrt{80}$

08. [illegible] (x_0, y_0) [illegible] (x_1, y_1) හා (x_2, y_2) [illegible]

[illegible] (x_1, y_1) [illegible]

$xx_1 + yy_1 + g(x + x_1) + f(y + y_1) + c = 0$

හා (x_2, y_2) [illegible]

$xx_2 + yy_2 + g(x + x_2) + f(y + y_2) + c = 0$

[illegible] (x_0, y_0) [illegible]

$x_0 x_1 + y_0 y_1 + g(x_0 + x_1) + f(y_0 + y_1) + c = 0$ —— ❶

$x_0 x_2 + y_0 y_2 + g(x_0 + x_2) + f(y_0 + y_2) + c = 0$ —— ❷

[illegible] ❶ හා ❷ [illegible]

$xx_0 + yy_0 + g(x + x_0) + f(y + y_0) + c = 0$

[illegible] (x_1, y_1) හා (x_2, y_2) [illegible]

[illegible] (x_1, y_1) හා (x_2, y_2) [illegible]

[illegible] (x_0, y_0) [illegible]

[illegible] (x_0, y_0) [illegible]

$xx_0 + yy_0 + g(x + x_0) + f(y + y_0) + c = 0$ [illegible]

$s = x^2 + y^2 + 2x + 6y + 1 = 0$ හා

$l = 4x + 3y - 5 = 0$ [illegible]

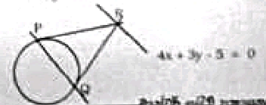
$s = 0$ [illegible] $(-1, -3)$ හා $r = \sqrt{1 + 9 - 1} = 3$

$(-1, -3)$ [illegible] $l = 0$ [illegible] d [illegible]

$$d = \frac{|4(-1) + 3(-3) - 5|}{\sqrt{16 + 9}} = \frac{|-18|}{5} = \frac{18}{5} = 3\frac{3}{5}$$

[illegible] $d > r$

∴ [illegible]



R [illegible] P [illegible] Q [illegible]

R, $4x + 3y - 5 = 0$ [illegible]

$$R \equiv \left[t,\ \frac{5-4t}{3}\right]$$ [illegible]

[illegible] $ax + by + c = 0$ [illegible] R [illegible]

$$xt + y\left(\frac{5-4t}{3}\right) + 1\,(x+t)$$
$$+ 3\left[y + \frac{5-4t}{3}\right] + 1 = 0$$

$3xt + y(5 - 4t) + 3(x + t) + 9y$
$+ 3(5 - 4t) + 3 = 0$

$x(3t + 3) + y(14 - 4t) + (18 - 9t) = 0$

[illegible]

$$\frac{3t+3}{a} = \frac{14-4t}{b} = \frac{18-9t}{c}$$

[illegible] $a = \dfrac{3b(t+1)}{2(7-2t)}$ [illegible] $c = \dfrac{9b(2-t)}{2(7-2t)}$

[illegible]

$$\frac{3b(t+1)}{2(7-2t)}\,x + by + \frac{9b(2-t)}{2(7-2t)} = 0$$

[illegible] $3b(t+1)x + 2b(7-2t)y + 9b(2-t) = 0$

$\Rightarrow 3(t+1)x + 2(7-2t)y + 9(2-t) = 0$

$\Rightarrow 3x + 14y + 18 + t(3x - 4y - 9) = 0$

[illegible] $3x + 14y + 18 = 0$ [illegible]

$3x - 4y - 9 = 0$ [illegible]

[illegible]

[illegible] $(1, -3/2)$ [illegible]

[illegible] $(1, -3/2)$ [illegible]

08. (a) $\sin^3 2x \cos 6x + \cos^3 2x \sin 6x$

$$= \left[\frac{3\sin 2x - \sin 6x}{4}\right]\cos 6x$$
$$+ \left[\frac{\cos 6x + 3\cos 2x}{4}\right]\sin 6x$$

$= \dfrac{1}{4}(3\sin 2x \cos 6x - \sin 6x\cos 6x$
$+ \cos 6x \sin 6x + 3\cos 2x \sin 6x)$

$= \dfrac{3}{4}(\sin 2x \cos 6x + \cos 2x \sin 6x)$

$= \dfrac{3}{4}\sin(2x + 6x)$

$= \dfrac{3}{4}\sin 8x$

$\sin^3 2x \cos 6x + \cos^3 2x \sin 6x = a$

$\Rightarrow \dfrac{3}{4}\sin 8x = a$ [illegible] $\sin 8x = \dfrac{4a}{3}$

[illegible]

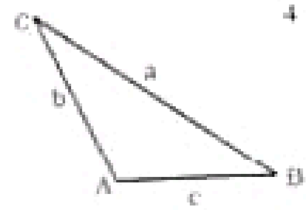
$-1 \le \sin 8x \le 1$ [illegible]

[illegible] $-1 \le \dfrac{4a}{3} \le 1$

[illegible]

$-\dfrac{3}{4} \le a \le \dfrac{3}{4}$ [illegible]

(b)



[illegible] A [illegible] C [illegible] a [illegible] c [illegible]

$A = 2C$ [illegible] $a = \dfrac{3}{2}c$ [illegible]

ABC [illegible]

$$\frac{\sin A}{a} = \frac{\sin C}{c}$$

[illegible] $$\frac{\sin 2C}{\frac{3}{2}c} = \frac{\sin C}{c}$$

$\Rightarrow 2\sin C\cos C = \dfrac{3}{2}\sin C$

$\sin C \neq 0$ [illegible] $2\cos C = \dfrac{3}{2}$

$\Rightarrow \cos C = \dfrac{3}{4} \Rightarrow C = \cos^{-1}\left(\dfrac{3}{4}\right)$

$A = 2C \Rightarrow B = \pi - 3C$

[illegible] $\dfrac{\sin B}{b} = \dfrac{\sin C}{c}$

$b = 10$ cm [illegible]

$$\frac{\sin(\pi - 3C)}{10} = \frac{\sin C}{c} \Rightarrow \frac{\sin 3C}{10} = \frac{\sin C}{c}$$

[illegible]

$$\frac{3\sin C - 4\sin^3 C}{10} = \frac{\sin C}{c}$$

$$\Rightarrow \frac{\sin C\,[3 - 4\sin^2 C]}{10} = \frac{\sin C}{c}$$

$\Rightarrow 3 - 4(1 - \cos^2 C) = \dfrac{10}{c}$ $(\sin C \neq 0)$

$\Rightarrow -1 + 4\cos^2 C = \dfrac{10}{c} \Rightarrow -1 + 4\cdot\dfrac{9}{16} = \dfrac{10}{c}$

$\Rightarrow \dfrac{5}{4} = \dfrac{10}{c} \Rightarrow c = 8$ cm $\Rightarrow a = \dfrac{3}{2}c$

$= \dfrac{3}{2}\times 8 = 12$ cm