

This image shows a scanned page from a mathematics textbook. The page contains several solved problems, some graphs, and a summary at the bottom.

Solved Problems:

- (i) $\alpha + \beta = -2 \Rightarrow \alpha\beta = 9$.
 $\alpha^2 - 1 = \gamma \Rightarrow \alpha^2 - 1 = 9 \Rightarrow \alpha^2 = 10$
 $\gamma + \delta = (\alpha^2 - 1) + (\beta^2 - 1)$
 $= \alpha^2 + \beta^2 - 2$
 $= (\alpha + \beta)^2 - 2|\alpha\beta| - 2$
 $= (-2)^2 - 2 \cdot 9 - 2$
 $= -16$
∴ $\gamma\delta = (\alpha^2 - 1)(\beta^2 - 1)$
 $= \alpha^2\beta^2 - (\alpha^2 + \beta^2) + 1$
 $= (\alpha\beta)^2 - ((\alpha + \beta)^2 - 2|\alpha\beta|) + 1$
 $= 81 - ((-2)^2 - 2 \cdot 9) + 1$
 $= 96$
- (ii) $x - y | x - y | = 0$
 $\Rightarrow x^2 - (y + 1)x + y^2 = 0$
 $x^2 + 16x + 96 = 0$
- (iii) $f(x) = k = x^2 + 2x + 9 = k$
 $\Rightarrow x^2 + 2x + 9 - k = 0$
मैंने यहाँ कोड करता हूँ कि यह क्षेत्र का क्षेत्र है।
 $\Delta = 0$
 $\Rightarrow 4 - 4(9 - k) = 0$
 $\Rightarrow 9 - k = 1 \Rightarrow k = 8$
- (iv) $\frac{1}{f(x)} = \frac{1}{x^2 + 2x + 9} = \frac{1}{(x+1)^2 + 8}$
यह ग्राफ एक वृत्तीय कीर्ति है।
बिन्दु $x = -1$ पर
वृत्तीय कीर्ति $= \frac{1}{8}$
- (v) $f(x) = \lambda x \cos x^2 + 2x + 9 = \lambda x$
 $\Rightarrow x^2 + (2 - \lambda)x + 9 = 0$
मैंने यहाँ कोड करता हूँ कि यह क्षेत्र का क्षेत्र है।
 $\text{क्षेत्र } (2 - \lambda)^2 - 36 < 0 \Rightarrow \lambda < 8$
 $\Rightarrow (\lambda + 4)(8 - \lambda) > 0$
 $\therefore \text{क्षेत्र } x < 0 \text{ और } x > 8$
 $= \{x \in \mathbb{R} : -4 < \lambda < 8\}$
- (vi) (a) $\text{मैंने कोड करता हूँ कि क्षेत्र}$
 $= \frac{12C_4}{4!8!}$

Graphs:

(a) A graph showing two intersecting lines, $y_1 = y_2 = x - 4/3$, and a parabola $y_2 = x^2 - (3x - a)$. The intersection point is marked as $A/3$.

(b) A graph showing two intersecting lines, $y_1 = y_2 = x - 4/3$, and a parabola $y_2 = x^2 - (3x - a)$. The intersection point is marked as $A/3$.

Summary:

प्रश्न संख्या - 2002 में से 1 से 6 तक के उत्तर।

प्रश्न संख्या - 2002 में से 7 से 12 तक के उत्तर।

प्रश्न संख्या - 2002 में से 13 से 18 तक के उत्तर।

प्रश्न संख्या - 2002 में से 19 से 24 तक के उत्तर।

प्रश्न संख्या - 2002 में से 25 से 30 तक के उत्तर।

$$\text{Soln:}$$

$$a + (b+3)x - 2$$

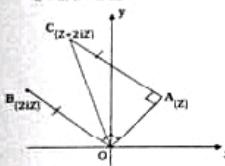
$$b = 3 \Rightarrow x = \frac{3}{3} = 1$$

$$a = 6 \times \frac{3}{3} - 2 = 6$$

$$\therefore \text{Ans}$$

$$\text{Ans b is correct}$$

$$a = 6, b = 3 \text{ is ok}$$



2 is angle between OA & OB since distance between OA & OB is $2|OA|$ & also OB is perpendicular to OA.
 $\theta + 2\theta = 180^\circ$
 OB is angle bisector of A since OA & OC are symmetric w.r.t. OB .
 $\hat{AOB} = \frac{\pi}{2} \Rightarrow \tan \hat{AOB} = \frac{AC}{OA}$
 $= \frac{2}{1} = 2$

(i) C is equidistant from OA & OB.

$$\arg(z) = \frac{\pi}{2} - \hat{AOB}$$

$$\text{Ans}$$

$$\frac{x}{x} = \tan\left(\frac{\pi}{2} - \hat{AOB}\right) = \cot \hat{AOB}$$

$$= \frac{1}{2}$$

$$\therefore x - 2y = 0$$

(ii) $y = 2x \Rightarrow \arg(z) = \tan^{-1}(2)$

$$\arg(z+2iz)$$

$$= \arg z(1+2i)$$

$$= \arg z + \arg(1+2i)$$

$$= \tan^{-1}(2) + \tan^{-1}(2)$$

$$= 2\tan^{-1}(2)$$

$$= 2\arg(z)$$

$$= \arg(z^2)$$

$$= x^2 \cdot OC \Rightarrow x=0$$



10

Given points z_1 and z_2 on circle $|z| = 2$
 $\arg z_1 = \frac{\pi}{2}$
 $\arg z_2 = \tan^{-1}(2) =$

$$\arg(z_1) + \tan^{-1}\left(\frac{1}{2}\right) = \arg(z_2)$$

$$\theta + \tan^{-1}\left(\frac{1}{2}\right) = \tan^{-1}(2)$$

$$\theta = \tan^{-1}(2) - \tan^{-1}\left(\frac{1}{2}\right)$$

$$= \tan^{-1}\left(\frac{3}{4}\right)$$

$$\text{Ans}$$

$$= \frac{1}{2} \cdot 0 \cdot r^2 = \frac{1}{2} \cdot 4^2 \tan^{-1}\left(\frac{3}{4}\right)$$

$$= 8 \tan^{-1}\left(\frac{3}{4}\right)$$

(iii) $y = e^{4x} \sin 3x$

$$= \frac{dy}{dx} = e^{4x} \cos 3x + \sin 3x \cdot 4e^{4x}$$

$$= 3e^{4x} \cos 3x + 4y = 0$$

$$\frac{d^2y}{dx^2} = 3e^{4x}(-3\sin 3x) + 3 \cos 3x \cdot 4e^{4x} + 4 \frac{dy}{dx}$$

$$= 3e^{4x}(-3\sin 3x + 12e^{4x} \cos 3x) + 4 \frac{dy}{dx}$$

$$= 3e^{4x} + 4 \frac{dy}{dx} - 16y + 4 \frac{dy}{dx}$$

$$= 8 \frac{dy}{dx} - 16y$$

$$= \frac{d^2y}{dx^2} - 8 \frac{dy}{dx} + 16y = 0$$

is homogeneous differential equation

$$\frac{d^2y}{dx^2} - 8 \frac{dy}{dx} + 25y = 0$$

$$\frac{d^2y}{dx^2} - 8 \frac{dy}{dx} + 25y = 0$$

$$x = 0 \Rightarrow \Phi(x)$$

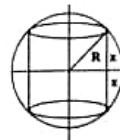
$$\left(\frac{dy}{dx}\right)_{x=0} = 3 \cdot e^0 \cos 0 + 0 = 3$$

$$\approx \pi \left(\frac{d^2y}{dx^2}\right)_{x=0} \cdot 8.3 + 25.0 = 0$$

$$\left(\frac{d^2y}{dx^2}\right)_{x=0} = 24$$

$$0 \cdot \left(\frac{d^3y}{dx^3}\right)_{x=0} = 8 \cdot 24 + 25 \cdot 3 = 0$$

$$\left(\frac{d^3y}{dx^3}\right)_{x=0} = 192 - 75 = 117$$



Required area R is calculated as $2\pi \int_R^R$ $V(x) dx$

$$V(x) = \pi(R^2 - x^2)2x$$

$$= 2\pi(R^2 - x^2)x$$

$$dV = 2\pi(R^2 - x^2)x(-2x)dx$$

$$= 2\pi(R^2 - 3x^2)x$$

$$= 6\pi \left[\frac{R^2}{3} - x^3\right]$$

$$= 6\pi \left[\frac{R^2}{3} - x\right] \left[\frac{R^2}{3} + x\right]$$

$$x > 0 \Rightarrow V(d) = 0.5\pi x = \frac{\pi}{3}R^3$$

$$x < \frac{R}{\sqrt{3}} \Rightarrow V(d) > 0$$

$$x > \frac{R}{\sqrt{3}} \Rightarrow V(d) < 0$$

$$\text{Ans} x = \frac{R}{\sqrt{3}} \Rightarrow V(d) = 0$$

$$\text{Ans value } V\left(\frac{R}{\sqrt{3}}\right)$$

$$= 2\pi \left[\frac{R^2}{3} - \frac{R^3}{3}\right] \frac{R}{\sqrt{3}}$$

$$= \frac{4\pi R^3}{3\sqrt{3}} \left(\frac{R^2}{3}\right)$$

$$= \frac{1}{\sqrt{3}}S$$

$$= \frac{4}{3}\pi R^3 \text{ enclosed volume of cylinder}$$

$$\text{Ans } \frac{1}{\sqrt{3}} \text{ value of } R \text{ is correct}$$

$$06. (a) I = \int_{-1}^2 \frac{x^3}{\sqrt{x^2 - 1}} dx \quad u = \sqrt{x^2 - 1}$$

Ans 0.5

$$\text{Ans } U^2 = x^2 - 1$$

$$\text{Ans } 2u du = 2x dx$$

$$\text{Ans}$$

$$I = \int_{-1}^2 \frac{(u^2 + 1)u}{u} du$$

$$= \int_{-1}^2 (u^2 + 1) du$$

$$= \left(\frac{u^3}{3} + u\right)_{-1}^2$$

$$= \left(\frac{8\sqrt{3}}{3} + \sqrt{3} - 0\right)$$

$$= 2\sqrt{3}$$

$$(b) \int_0^1 x \tan^{-1} x dx$$

$$= \int_0^1 \tan^{-1} x \cdot \frac{d}{dx} \left(\frac{x^2}{2}\right) dx$$

$$= \left[\tan^{-1} x \cdot \frac{x^2}{2}\right]_0^1 - \frac{1}{2} \int_0^1 x^2 \cdot \frac{1}{1+x^2} dx$$

$$= \left[\frac{\pi}{4} \cdot \frac{1}{2} - 0\right] -$$

$$\frac{1}{2} \int_0^1 \left[\frac{1-x^2}{1+x^2} - \frac{1}{1+x^2} \right] dx$$

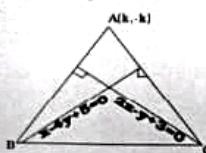
$$= \frac{1}{2} \cdot \frac{1}{2} \left[x \right]_0^1 + \frac{1}{2} \left[\tan^{-1} x \right]_0^1$$

$$= \frac{1}{4} + \frac{1}{2} \cdot \frac{\pi}{4}$$

$$= \frac{1}{4} + \frac{\pi}{8}$$

$$= \frac{1}{4} + \frac{\pi}{8}$$

Ques. No. (odd nos.) Date :- 2002 marks		Ans. Keys
<u>Ques. no. 1</u>		
$\frac{5x-4}{(2+x)(1-x+x^2)} = \frac{A}{2+x} + \frac{Bx+C}{1-x+x^2}$ $\Rightarrow A = -2, B = 2, C = -1$		
$\therefore \int \frac{5x-4}{(1-x+x^2)(2+x)} dx$ $= \int \frac{2x-1}{(1-x+x^2)} dx - 2 \int \frac{dx}{x+2}$ $= [\ln 1-x+x^2]_1^2 - 2 [\ln x+2]_1^2$ $= (m \cdot 3 - m \cdot 1) - 2(m \cdot 4 - m \cdot 3)$ $= 3 \cdot 1 \cdot 3 - 2 \cdot 4$ $= 27/16$		
$AB, 2x+y+3=0 \text{ द्वारा दिया गया है}$ $AB, x+2y+p=0 \text{ द्वारा दिया गया है}$ $AB, x-y-k=0 \text{ द्वारा दिया गया है}$ $k-2x+p=0 \Rightarrow p=k$ $AB \text{ द्वारा दिया गया } x+2y+k=0 \text{ है}$ $AC, x-4y+5=0 \text{ द्वारा दिया गया है}$ $AC \text{ द्वारा दिया गया } 4x+y+q=0 \text{ द्वारा दिया गया है}$ $A(0, -k) \text{ को } x=0 \text{ पर लें}$ $4k+k+q=0 \Rightarrow q=-5k$ $-AC \text{ द्वारा दिया गया } 4x+y-3k=0 \text{ है}$ $2x+y+3=0 \Rightarrow 4x+2y+6=0 \text{ है}$ $\text{संतुष्टि दिया गया है}$ $C = \left[\frac{k}{2}, -k+2 \right]$ $x-4y+5=0 \Rightarrow x+2y-k=0 \text{ है}$ $\text{संतुष्टि दिया गया है}$ $\left[(-2k+5), 5-k \right]$		



Exercice 1 2002 exercice 1

AB, $2x + y + 3 = 0$ est une droite
 AC, $x + 2y + p = 0$ est une droite
 so A(k, -k) milie de AB et de AC
 $-2k + p = k$

AB \parallel AC \Leftrightarrow $k + 2y + k = 0 \Rightarrow$
 AC, $x - 4y + 5 = 0 \Leftrightarrow$ $k = -2$
 AC \parallel BC \Leftrightarrow $4x + y + q = 0$ est une droite
 A(0, -k) milie de AC et de BC
 $4k - k + q = 0 \Rightarrow q = -3k$
 AC \parallel BC \Leftrightarrow $4x + y - 3k = 0 \parallel$

$2x + y + 3 = 0 \Leftrightarrow 4x + y - 3k = 0 \in$ B
 Milie de AB et de BC

$C = \left[\frac{k-1}{2}, k+2 \right]$

$x - 4y + 5 = 0 \Leftrightarrow x - 2y - k = 0$ B est
 Milie de AC et de BC

B = $\left[\frac{(2k+5)}{3}, \frac{5-k}{6} \right]$

எனவே $u_1 = 0 = u_2 = 0$ என்றால் கூடும் வகு விடை
 கீழ்க்கண்ட சம்பந்தமாக கொண்டு வரும் வகு விடை
 என்றால்

$$a_1x_0 + b_1y_0 + c_1 = 0 \Rightarrow 0 = 0$$

$$a_2x_0 + b_2y_0 + c_2 = 0 \Rightarrow 0 = 0$$

 எனவே $u_1 = \lambda u_2 = a_1x + b_1y + c_1$

$$\Rightarrow \lambda(a_2x + b_2y + c_2) = 0 = 0 \text{ என்றால்}$$

$$= (\lambda_1 + \lambda_2)x + (b_1 + b_2)y + (c_1 + \lambda_2c_2) = 0$$

 அதே போல் கூடும் வகு விடை கீழ்க்கண்ட வகு விடை
 என்றால் கொண்டு வரும் வகு விடை என்றால்
 எனவே $u_1 = 0 = u_2 = 0$ என்றால் $\lambda = 0$ என்றால் கூடும் வகு

$$a_1x_0 + b_1y_0 + c_1 = 0 \Rightarrow 0 = 0$$

$$a_2x_0 + b_2y_0 + c_2 = 0 \Rightarrow 0 = 0$$

 எனவே $u_1 = \lambda u_2 = a_1x + b_1y + c_1$

$$\Rightarrow \lambda(a_2x + b_2y + c_2) = 0 = 0 \text{ என்றால்}$$

$$\begin{aligned} \text{समान्तर श्रेणी का } n\text{-वां पद } G_n &= (k, \bar{y}) \text{ है।} \\ \text{तो } G_n &= \frac{1}{3} \left[k + \left(\frac{2(k+5)}{3} \right) + \left(\frac{k-1}{2} \right) \right] \\ &= \frac{6k + 10 + 3k - 3}{18} \\ &= \frac{5k + 13}{18} \\ \text{यह } 5k + 13 &= 0 \\ \text{का } 5k &= -13 \\ \text{तो } k &= -\frac{13}{5} \\ \text{अब } x &= \frac{1}{3} \left[k + \left(\frac{2(-k-5)}{3} \right) + \left(\frac{-k+21}{2} \right) \right] \\ &= \frac{-6k - 10 - k + 10k + 12}{18} \\ &= \frac{17 - k}{18} \\ \text{तो } x &= \frac{17 - (-\frac{13}{5})}{18} \\ &= \frac{55 + 13}{18} = \frac{68}{18} = \frac{34}{9} \\ \text{तो } x &= \frac{34}{9} \end{aligned}$$

Q4. చూడాలి ద్వారా దొరు వా పాపికి కోణాలను $C_1 = C_2$ కంటి కాగి $C_1C_2 = |r_1 - r_2|$ కంటి.

$$\begin{aligned} & \text{cos}\theta = \frac{\mathbf{u}_1 \cdot \mathbf{u}_2}{\|\mathbf{u}_1\| \|\mathbf{u}_2\|} \\ & = \frac{(g_1 - g_2)^2 + f_1^2 + c_1^2 - g_2^2 - f_2^2 - c_2^2}{\sqrt{(g_1 - g_2)^2 + f_1^2 + c_1^2} \sqrt{g_2^2 + f_2^2 + c_2^2}} / \\ & \text{where } \mathbf{u}_1 = (g_1, f_1, c_1) \text{ and } \mathbf{u}_2 = (g_2, f_2, c_2) \\ & \text{and } \mathbf{u}_1 \cdot \mathbf{u}_2 = g_1g_2 + f_1f_2 + c_1c_2 \end{aligned}$$

$$\begin{aligned} & \text{Solve: } \\ & (I_1 - I_2)x - (E_1 - E_2)y + I_1E_2 - I_2E_1 = 0 \\ & \text{and:} \\ & x_o^2 + y_o^2 + 2g_x x_o + 2I_y y_o + c_1 = 0 \\ & \quad = 0 \\ & x_o^2 + y_o^2 + 2g_x x_o + 2I_y y_o + c_2 = 0 \\ & \Rightarrow 2(E_1 - E_2)y_o + 2(I_1 - I_2)x_o + c_1 - c_2 = 0 \\ & \text{and: } \\ & 2(E_1 - E_2)x_o + 2(I_1 - I_2)y_o + c_1 - c_2 = 0 \\ & \Rightarrow c_1 = c_2 \end{aligned}$$

$$\begin{aligned} & \text{Circles intersect externally} \\ & x^2 + y^2 - 2x + 4y = 0 \text{ and} \\ & x^2 + y^2 - 10x + 20 = 0 \text{ are } 50 \\ & C_1 C_2 = \sqrt{(-1+5)^2 + (2-0)^2} \\ & = \sqrt{20} = 2\sqrt{5} \\ & r_1 = \sqrt{(-1)^2 + 2^2 - 0} = \sqrt{5} \\ & r_2 = \sqrt{5^2 - 20} = \sqrt{5} \end{aligned}$$

Ques 2003 Ans

Ques I Ans Page 5

$x_1 + x_2 = 2\sqrt{5}$

 $\therefore C_1 C_2 = x_1 + x_2$
 \therefore दोनों दिल्ली दिल्ली का।

दोनों दिल्ली दिल्ली का तो $C_1 C_2$ जूहा दिल्ली का।

दोनों दिल्ली दिल्ली का तो $C_1 C_2$ जूहा दिल्ली का।

दोनों दिल्ली दिल्ली का तो $A = \begin{bmatrix} 1+3 & 1+3+1 \\ 2 & 2 \end{bmatrix}$

 $= (3, -1)$

P(x, y) का दोनों दिल्ली का दिल्ली।

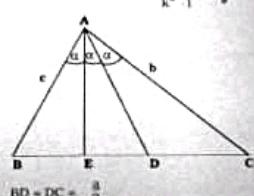
अब P का दोनों दिल्ली का दिल्ली $\sqrt{x^2 + y^2} - 2x + 4y$ का दोनों दिल्ली का दिल्ली $\sqrt{x^2 + y^2} - 10x + 20$ का दिल्ली

 $\sqrt{x^2 + y^2} - 2x + 4y$
 $= k \sqrt{x^2 + y^2} - 10x + 20$
 $\Rightarrow x^2 + y^2 - 2x + 4y$
 $= k^2 (x^2 + y^2) - 10x + 20$
 $\Rightarrow (k^2 - 1)x^2 + (k^2 - 1)y^2 - 10x^2 - 2x$
 $- 4y + 20k^2 = 0$
 $(x^2 + y^2)(k^2 - 1) - 10x^2 - 2x$
 $- 4y + 20k^2 = 0$

$k^2 - 1 \neq 0$ अतः P B का A का दोनों दिल्ली का दिल्ली।

अतः दोनों दिल्ली का

 $x^2 + y^2 - \frac{10x^2 - 2x}{k^2 - 1} - \frac{4}{k^2 - 1} y$
 $+ \frac{20k^2}{k^2 - 1} = 0$



$$\frac{BD}{\sin 2\alpha} = \frac{AB}{\sin ADB}$$

$$\Rightarrow \sin ADB = \frac{AB}{BD} \sin 2\alpha$$

ADC A तो 2 के बराबर होता है

$$\frac{AC}{\sin ADC} = \frac{DC}{\sin \alpha}$$

$$\Rightarrow \sin ADC = \frac{AC}{DC} \sin \alpha$$

$$\sin ADB = \sin ADC$$

$$\Rightarrow \frac{AB}{DC} \sin \alpha = \frac{BD}{DC} \sin 2\alpha$$

$$\Rightarrow AC \sin \alpha = AB \cdot 2 \sin \alpha \cos \alpha \quad (\because DC = BD \text{ तो})$$

$$\Rightarrow \cos \alpha = \frac{AC}{2AB} = \frac{b}{2c}$$

$$\Rightarrow \cos \frac{\alpha}{2} = \frac{b}{2c} /$$

$$DE : EB = 1 : k \Rightarrow DE = \frac{1}{1+k} BD \Rightarrow$$

$$EB = \frac{k}{1+k} BD$$

$$\Rightarrow DE = \frac{a}{2(1+k)} \Rightarrow EB = \frac{ak}{2(1+k)}$$

में से

$$EC + DE + DC = \frac{a(2+k)}{2(1+k)}$$

AEC A तो 2 के बराबर होता है

$$\frac{EC}{\sin 2\alpha} = \frac{AC}{\sin AEC}$$

$$\Rightarrow \sin AEC = \frac{AC}{EC} \sin 2\alpha$$

दोनों AEB A तो

$$\frac{BE}{\sin \alpha} = \frac{AB}{\sin AEB}$$

$$\Rightarrow \sin AEB = \frac{AB}{BE} \sin \alpha$$

$$\sin AEC = \sin AEB$$

$$\Rightarrow \frac{AC}{EC} \sin 2\alpha = \frac{AB}{BE} \sin \alpha$$

$$\Rightarrow \cos \alpha = \frac{1}{2} \frac{AB \cdot EC}{BE \cdot AC}$$

$$= \frac{1}{2} \frac{2c(1+k)}{ak} \frac{a(2+k)}{2b(1+k)}$$

$$= \frac{(1+k)^2}{2bk} /$$

$$k = 1 \text{ तो } \cos \alpha = \frac{3c}{2b} = \frac{b}{2c} \Rightarrow b^2 = 3c^2$$

$$\Rightarrow \left(\frac{b}{2c}\right)^2 = \frac{3}{4}$$

$$\Rightarrow \cos \alpha = \frac{b}{2c} = \frac{\sqrt{3}}{2} \Rightarrow \alpha = 30^\circ$$

$$\therefore A = 3\alpha = 90^\circ$$

$$k = 2 \text{ तो } \cos \alpha = \frac{4c}{4b} = \frac{b}{2c} = b^2 = 2c^2$$

$$\Rightarrow \left(\frac{b}{2c}\right)^2 = \frac{1}{2}$$

$$\Rightarrow \cos \alpha = \frac{b}{2c} = \frac{1}{\sqrt{2}} \Rightarrow \alpha = 45^\circ$$

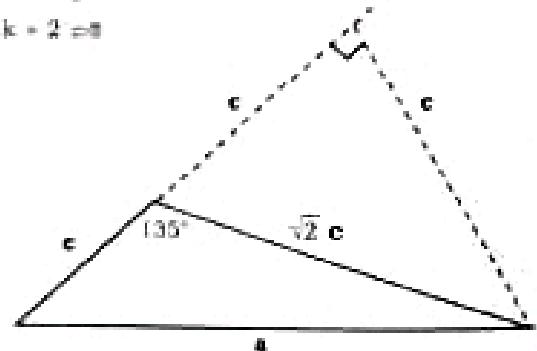
$$\therefore A = 135^\circ /$$

$$k = 1 \text{ तो}$$

$$a^2 = b^2 + c^2 = 3c^2 + c^2 = 4c^2 \Rightarrow c = \frac{a}{2}$$

$$b = \frac{\sqrt{3}}{2} a$$

$$k = 2 \text{ तो}$$



$$a^2 = (AC)^2 + (AB)^2 + 2(AB)(AC) \cos 45^\circ$$

$$= 2c^2 + c^2 + 2\sqrt{2}c^2 \frac{1}{\sqrt{2}}$$

$$\Rightarrow c^2 = \frac{a^2}{5}$$

$$\therefore c = \frac{a}{\sqrt{5}}, b = \sqrt{\frac{2}{5}} a /$$

*** ***