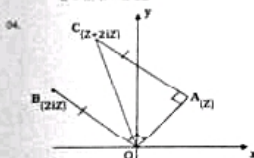


21. (i) $\lim_{x \rightarrow 0} f(x) = 0$ $\forall \epsilon > 0$ $\exists \delta > 0$ s.t.

$$\alpha + \beta = 2 \Rightarrow \alpha\beta = 9 \Rightarrow \alpha = 3, \beta = 3$$

04. $a = (b+3)x - 2$
 $b = 3 \sin x = \frac{4}{3}$ වේ
 $a = 6 \times \frac{4}{3} - 2 = 6$
එනම්
අංක b හි අගයය
 $a = 6, b = 3$ වේ



2 හි පරිදිම අගයය OA හි දිග 2 වන බැවින් OB හි දිග 2 වේ. OC හි දිග $2\sqrt{2}$ වේ. OB හි දිග 2 වේ. OC හි දිග $2\sqrt{2}$ වේ. OB හි දිග 2 වේ. OC හි දිග $2\sqrt{2}$ වේ.

$\angle AOB = \frac{\pi}{2}$ වේ. $\angle AOC = \frac{\pi}{4}$ වේ.
 $\angle AOB = \frac{\pi}{2}$ වේ. $\angle AOC = \frac{\pi}{4}$ වේ.

(i) C ලක්ෂ්‍යයේ ස්ථාන සොයන්න.
 $\arg(z) = \frac{\pi}{2} - \angle AOC$
එනම්
 $\frac{\pi}{2} = \tan^{-1}\left(\frac{y}{x}\right) - \angle AOC$
 $\frac{\pi}{2} = \tan^{-1}\left(\frac{y}{x}\right) - \frac{\pi}{4}$
 $\frac{3\pi}{4} = \tan^{-1}\left(\frac{y}{x}\right)$
 $\frac{y}{x} = \tan\left(\frac{3\pi}{4}\right) = -1$
 $y = -x$

(ii) $y = 2x$ වේ. $\arg(z) = \tan^{-1}(2)$
එනම්
 $\arg(z + 2i) = \arg(z) + \arg(1 + 2i)$
 $= \tan^{-1}(2) + \tan^{-1}(2)$
 $= 2 \tan^{-1}(2)$
 $= 2 \arg(z)$
 $= \arg(z^2)$
 $= \arg(z^2)$



05.

අනෙකුත් පරිදිම අගයය OA හි දිග 2 වන බැවින් OB හි දිග 2 වේ. OC හි දිග $2\sqrt{2}$ වේ. OB හි දිග 2 වේ. OC හි දිග $2\sqrt{2}$ වේ.

$\arg(z_2) = \tan^{-1}\left(\frac{y}{x}\right) = \tan^{-1}\left(\frac{2}{2}\right) = \frac{\pi}{4}$
එනම් $\arg(z_2) = \frac{\pi}{4}$ වේ.
 $\beta - \alpha = 0$ වේ.
 $\tan \theta = \frac{\tan \beta - \tan \alpha}{1 + \tan \beta \tan \alpha}$
 $= \frac{\tan \frac{\pi}{4} - \tan \frac{\pi}{4}}{1 + \tan \frac{\pi}{4} \tan \frac{\pi}{4}} = 0$
 $\theta = 0$

$\theta = \tan^{-1}\left(\frac{3}{4}\right)$
එනම් $\theta = \tan^{-1}\left(\frac{3}{4}\right)$ වේ.
 $\theta = \tan^{-1}\left(\frac{3}{4}\right)$

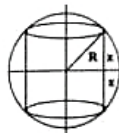
05. (a) $y = e^{4x} \sin 3x$
 $\frac{dy}{dx} = e^{4x} (3 \cos 3x + \sin 3x) 4e^{4x}$
 $= 3e^{8x} \cos 3x + 4e^{8x} \sin 3x$
 $\frac{d^2y}{dx^2} = 3e^{8x} (-3 \sin 3x) + 4e^{8x} (3 \cos 3x)$
 $= -9e^{8x} \sin 3x + 12e^{8x} \cos 3x$
 $= -9y + 12e^{8x} \cos 3x$
 $= -9y + 12e^{8x} \cos 3x$

$\frac{d^2y}{dx^2} - 8 \frac{dy}{dx} + 25y = 0$
 $\frac{d^2y}{dx^2} - 8 \frac{dy}{dx} + 25y = 0$

පිටුව 2

$x = 0$ වේ. $\frac{dy}{dx} = 3e^0 \cos 0 + 0 = 3$
 $\frac{dy}{dx} = 3e^0 \cos 0 + 0 = 3$
 $\frac{d^2y}{dx^2} = 8.3 + 25.0 = 0$
 $\frac{d^2y}{dx^2} = 24$
 $\frac{d^3y}{dx^3} = 8 \times 24 + 25 \times 3 = 0$
 $\frac{d^4y}{dx^4} = 192 - 75 = 117$

06.



එනම් $R \cos \theta = R \cos \theta$ වේ. $R \sin \theta = R \sin \theta$ වේ. $R \cos \theta = R \cos \theta$ වේ. $R \sin \theta = R \sin \theta$ වේ.

$V(x) = 2\pi (R^2 - x^2) x$
 $= 2\pi (R^3 - R x^3)$
 $= 6\pi \left[\frac{R^3}{3} - \frac{R x^3}{3} \right]$
 $= 2\pi \left[\frac{R^3}{3} - \frac{R x^3}{3} \right]$
 $x > 0$ වේ. $V(x) = 0$ වේ. $x = \frac{R}{\sqrt{3}}$
 $x < \frac{R}{\sqrt{3}}$ වේ. $V(x) > 0$
 $x > \frac{R}{\sqrt{3}}$ වේ. $V(x) < 0$

එනම් $x = \frac{R}{\sqrt{3}}$ වේ. $V(x)$ හි අගයය $\frac{2\pi R^3}{3\sqrt{3}}$ වේ.
 $V(x) = \frac{2\pi R^3}{3\sqrt{3}}$

$\frac{1}{\sqrt{3}} S$
 $S = \frac{4}{3} \pi R^3$ වේ. $S = \frac{4}{3} \pi R^3$ වේ. $S = \frac{4}{3} \pi R^3$ වේ.

06. (a) $I = \int \frac{x^3}{\sqrt{x^2-1}} dx$ $u = \sqrt{x^2-1}$
 $u^2 = x^2 - 1$
 $2u du = 2x dx$
 $x dx = u du$
 $I = \int \frac{u^3}{u} u du$
 $= \int u^3 du$
 $= \frac{u^4}{4} + C$
 $= \frac{(x^2-1)^2}{4} + C$

(b) $I = \int x \tan^{-1} x dx$
 $= \int \tan^{-1} x \cdot \frac{d}{dx} \left(\frac{x^2}{2} \right) dx$
 $= \left[\tan^{-1} x \cdot \frac{x^2}{2} \right]_0^1 - \int_0^1 \frac{x^2}{2} \cdot \frac{1}{1+x^2} dx$
 $= \left[\frac{x^2}{2} \tan^{-1} x \right]_0^1 - \frac{1}{2} \int_0^1 \frac{x^2}{1+x^2} dx$
 $= \frac{1}{2} \left[\frac{x^2}{2} \tan^{-1} x \right]_0^1 - \frac{1}{2} \left[\frac{x^2}{2} \tan^{-1} x \right]_0^1$
 $= \frac{1}{2} \left[\frac{x^2}{2} \tan^{-1} x \right]_0^1 - \frac{1}{2} \left[\frac{x^2}{2} \tan^{-1} x \right]_0^1$

පිටුව 3

ABD Δ 2nd year exam

$$\frac{BD}{\sin 2\alpha} = \frac{AB}{\sin \hat{ADB}}$$

$$\Rightarrow \sin \hat{ADB} = \frac{AB}{BD} \sin 2\alpha$$

ADC Δ 2nd year exam

$$\frac{AC}{\sin \hat{ADC}} = \frac{DC}{\sin \alpha}$$

$$\Rightarrow \sin \hat{ADC} = \frac{AC}{DC} \sin \alpha$$

$$\sin \hat{ADB} = \sin \hat{ADC}$$

$$\Rightarrow \frac{AB}{DC} \sin 2\alpha = \frac{AC}{DC} \sin \alpha$$

$$\Rightarrow AC \sin \alpha = AB \cdot 2 \sin \alpha \cos \alpha$$

($\because DC = BD$ as $\hat{ADB} = \hat{ADC}$)

$$\Rightarrow \cos \alpha = \frac{AC}{2AB} = \frac{b}{2c}$$

$$\Rightarrow \cos \frac{A}{2} = \frac{b}{2c}$$

$$DE : EB = 1 : k \Rightarrow DE = \frac{1}{1+k} BD \Rightarrow$$

$$EB = \frac{k}{1+k} BD$$

$$\Rightarrow DE = \frac{a}{2(1+k)} \Rightarrow EB = \frac{ak}{2(1+k)}$$

and

$$EC = DE + DC = \frac{a(2+k)}{2(1+k)}$$

AEC Δ 2nd year exam

$$\frac{EC}{\sin 2\alpha} = \frac{AC}{\sin \hat{AEC}}$$

$$\Rightarrow \sin \hat{AEC} = \frac{AC}{EC} \sin 2\alpha$$

and AEB Δ 2nd

$$\frac{BE}{\sin \alpha} = \frac{AB}{\sin \hat{AEB}}$$

$$\Rightarrow \sin \hat{AEB} = \frac{AB}{BE} \sin \alpha$$

$$\sin \hat{AEC} = \sin \hat{AEB}$$

$$\Rightarrow \frac{AC}{EC} \sin 2\alpha = \frac{AB}{BE} \sin \alpha$$

$$\Rightarrow \cos \alpha = \frac{1}{2} \frac{AB}{BE} \frac{EC}{AC}$$

$$= \frac{1}{2} \frac{2c(1+k)}{ak} \frac{a(2+k)}{2b(1+k)}$$

$$= \frac{(2+k)c}{2bk}$$

$$k=1 \Rightarrow \cos \alpha = \frac{3c}{2b} = \frac{b}{2c} \Rightarrow b^2 = 3c^2$$

$$\Rightarrow \left(\frac{b}{2c}\right)^2 = \frac{3}{4}$$

$$\Rightarrow \cos \alpha = \frac{b}{2c} = \frac{\sqrt{3}}{2} \Rightarrow \alpha = 30^\circ$$

$$\therefore A = 3\alpha = 90^\circ$$

$$k=2 \Rightarrow \cos \alpha = \frac{4c}{4b} = \frac{b}{2c} \Rightarrow b^2 = 2c^2$$

$$\Rightarrow \left(\frac{b}{2c}\right)^2 = \frac{1}{2}$$

$$\Rightarrow \cos \alpha = \frac{b}{2c} = \frac{1}{\sqrt{2}} \Rightarrow \alpha = 45^\circ$$

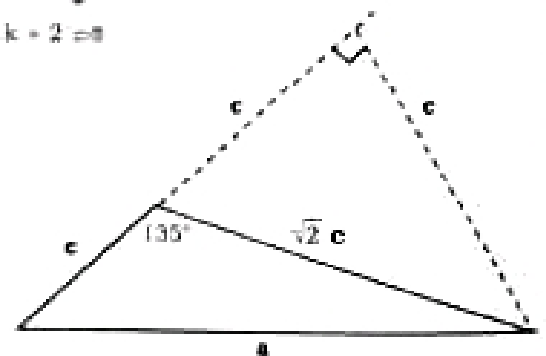
$$\therefore A = 135^\circ$$

$$k=1 \Rightarrow$$

$$a^2 = b^2 + c^2 = 3c^2 + c^2 = 4c^2 \Rightarrow c = \frac{a}{2}$$

$$b = \frac{\sqrt{3}}{2} a$$

$$k=2 \Rightarrow$$



$$a^2 = (AC)^2 + (AB)^2 + 2(AB)(AC) \cos 45^\circ$$

$$= 2c^2 + c^2 + 2\sqrt{2}c^2 \cdot \frac{1}{\sqrt{2}}$$

$$= 3c^2$$

$$\Rightarrow c^2 = \frac{a^2}{3}$$

$$\therefore c = \frac{a}{\sqrt{3}}, b = \sqrt{\frac{2}{3}} a$$
