

01. (i) $P(x) = (\lambda - 2)x^2 - 3(\lambda + 2)x + 6$
 $\lambda = 2$ දී $P(x) = -12x + 12 \neq 0$
 $\lambda = 2$ නිසා $x = 3$ දී $P(3) = -24 < 0$
 එබැවින් $\lambda = 2$ දී $P(x)$ සාධාරණ x සඳහා $P(x)$ යනු ධන නොවේ.
 $\lambda \neq 2$ දී $\lambda - 2 < 0$ දී $P(x)$ සඳහා $P(x)$ යනු ධන λ වනුයේ නොවේ.
 $\lambda - 2 > 0$ නිසා
 $\Delta_x = 9(\lambda + 2)^2 - 4(\lambda - 2)6 < 0$ දී
 එවිට $\forall x \in \mathbb{R}$ සඳහා $P(x) > 0$ වේ.
 එවිට $\lambda \neq 2$ නිසා $\Delta = 9(\lambda^2 + 4\lambda + 4) - 24\lambda^2 + 48\lambda < 0$
 $\Rightarrow \lambda > 2$ නිසා
 $-15\lambda^2 + 84\lambda + 36 < 0$
 $\Rightarrow \lambda > 2$ නිසා $5\lambda^2 - 28\lambda - 12 > 0$
 $\Rightarrow \lambda > 2$ නිසා $(5\lambda + 2)(\lambda - 6) > 0$
 $\Rightarrow \lambda > 2$ නිසා $\lambda < -\frac{2}{5}$ සහ $\lambda > 6$
 $\Rightarrow \lambda > 6$
 $\therefore x \in \mathbb{R}$ සඳහා $P(x) > 0$ යනු $\lambda > 6$ වේ.
 අනෙක් අංශයේ සඳහා $\lambda = 7$ වේ.

- (ii) $P(x) = 0$ හි පෘථිවික සාධාරණ සඳහා $\lambda \neq 2$ නිසා $\Delta > 0$ විය යුතුය.
 එවිට $\lambda \neq 2$ නිසා $(5\lambda + 2)(\lambda - 6) < 0$
 $\Rightarrow \lambda \neq 2$ නිසා $-\frac{2}{5} < \lambda < 6$
 එවිට $\lambda \in \left(-\frac{2}{5}, 2\right) \cup (2, 6)$

- (iii) $P(x) = 0$ හි පෘථිවික සාධාරණ සඳහා α හි β යනු $\alpha > \beta$ යනු සිදුවේ.
 $\alpha + \beta = \frac{3(\lambda + 2)}{(\lambda - 2)}$ නිසා $\alpha\beta = \frac{6\lambda}{\lambda - 2}$
 $\Rightarrow \alpha - \beta = 1$ වේ.
 $(\alpha - \beta)^2 = (\alpha + \beta)^2 - 4\alpha\beta$
 $1 = \frac{9(\lambda + 2)^2}{(\lambda - 2)^2} - \frac{24\lambda}{\lambda - 2}$
 $\Rightarrow 9(\lambda - 2)^2 = 9(\lambda + 2)^2 - 24\lambda(\lambda - 2)$
 $\Rightarrow 9\lambda^2 - 36\lambda + 36 = 9\lambda^2 + 36\lambda + 36 - 24\lambda^2 + 48\lambda$
 $\Rightarrow 24\lambda^2 - 120\lambda = 0$
 $\Rightarrow \lambda^2 - 5\lambda = 0$
 $\Rightarrow \lambda(\lambda - 5) = 0$
 $\therefore \lambda = 0$ සහ $\lambda = 5$

02. (a)

පෘථිවික සඳහා

පෘථිවික සඳහා

(a) 5, 1, 1, 1	$\frac{6!}{5!(1!)^3 \cdot 3!} = 56$
(b) 4, 2, 1, 1	$\frac{6!}{4!2!(1!)^2 \cdot 2!} = 420$
(c) 3, 3, 1, 1	$\frac{6!}{(3!)^2(1!)^2 \cdot 2! \cdot 2!} = 280$
(d) 3, 2, 2, 1	$\frac{6!}{3!(2!)^2 \cdot 1! \cdot 2!} = 840$
(e) 2, 2, 2, 2	$\frac{6!}{(2!)^4 \cdot 4!} = 105$

$\therefore$ සියලුම පෘථිවික සාධාරණ සඳහා

$$= 56 + 420 + 280 + 840 + 105$$

$$= 1701$$

$$(iv) {}^nC_{r+1} + {}^nC_r$$

$$= \frac{n!}{(n-r-1)!(r+1)!} + \frac{n!}{(n-r)!(r)!}$$

$$= \frac{n!(n-r)}{(n-r)!(r+1)!} + \frac{n!(r+1)}{(n-r)!(r+1)!}$$

$$= \frac{n!(n-r+r+1)}{(n-r)!(r+1)!}$$

$$= \frac{(n+1)!}{(n+1-r-1)!(r+1)!}$$

$$= {}^{n+1}C_{r+1}$$

අනෙක් ප්‍රවේශයට අනුව

$$2003C_r = 2004C_{r+1} - 2003C_{r+1}$$

$$2004C_r = 2005C_{r+1} - 2004C_{r+1}$$

$$2005C_r = 2006C_{r+1} - 2005C_{r+1}$$

$$2013C_r = 2014C_{r+1} - 2013C_{r+1}$$

$$2003C_r + 2004C_r + \dots + 2013C_r = 2014C_{r+1} - 2003C_{r+1}$$

$$(v) (a) F(n) : 8(n+1)! > 2^{n+1}(n+2)$$

නිසා සත්‍යය

$$n = 1$$

$$L.H.S = 8 \times 2! = 16$$

$$R.H.S = 4 \times 3 = 12$$

එබැවින් සත්‍යය ප්‍රකාශනයයි.

$$\begin{aligned}
 (b) \quad & \sin 2x + \sin 4x + \sin 6x \\
 &= \sin 4x + (\sin 2x + \sin 6x) \\
 &= \sin 4x + 2\sin 4x \cos 2x \\
 &= (1 + 2\cos 2x) \sin 4x \\
 &= \sin x (\sin 2x + \sin 4x + \sin 6x) \\
 &= \sin x (1 + 2\cos 2x) \sin 4x \\
 &= \sin 4x (\sin x + 2\sin x \cos 2x) \\
 &= \sin 4x (\sin x + \sin 3x - \sin x) \\
 &= \sin 3x \sin 4x
 \end{aligned}$$

$$x = \frac{\pi}{12} \quad \text{1000 1000}$$

$$\sin \frac{\pi}{12} \left(\frac{1}{2} + \frac{\sqrt{3}}{2} + 1 \right) = \frac{1}{\sqrt{2}} \times \frac{\sqrt{3}}{2}$$

$$\left(\frac{3 + \sqrt{3}}{2} \right) \sin \frac{\pi}{12} = \frac{\sqrt{3}}{2\sqrt{2}}$$

$$\sin \frac{\pi}{12} = \frac{2\sqrt{3}}{2\sqrt{2}(3 + \sqrt{3})}$$

$$\sin \frac{\pi}{12} = \frac{\sqrt{3}(3 - \sqrt{3})}{\sqrt{2}(9 - 3)}$$

$$= \frac{3(\sqrt{3} - 1)}{6\sqrt{2}}$$

$$= \frac{\sqrt{6} - \sqrt{2}}{4}$$

1000 1000 $\therefore$ ABC 1000 1000 1000

$$\frac{\sin A}{a} = \frac{\sin B}{b} = \frac{\sin C}{c} \quad \text{1000}$$

$$\frac{\sin A}{a} = \frac{\sin B}{b} = \frac{\sin C}{c} = k$$

1000 1000

$$a = b + \lambda c \quad \text{1000}$$

$$\Rightarrow \sin A = \sin B + \lambda \sin C$$

$$\begin{aligned}
 \Rightarrow \lambda \sin C &= \sin A - \sin B \\
 &= \sin (B + C) - \sin B \\
 &(\because A + B + C = \pi)
 \end{aligned}$$

$$\begin{aligned}
 \Rightarrow 2\lambda \sin \frac{C}{2} \cos \frac{C}{2} &= 2\sin \frac{C}{2} \cos \frac{(2B + C)}{2} \\
 \Rightarrow \lambda \cos \frac{C}{2} &= \cos \left(B + \frac{C}{2} \right)
 \end{aligned}$$