

01. (i) $P(x) = (\lambda - 2)x^2 - 3(\lambda + 2)x + 6$
 $\lambda = 2$ උදා: $P(x) = -12x + 12 < 0$
 $\lambda = 2$ නොවන නිසා $x = 3$ උදා: $P(3) = -24 < 0$
 එසේ $\lambda = 2$ උදා: සෑම x සඳහාම $P(x)$ ඍණ වේ.
 $\lambda \neq 2$ උදා: $\lambda - 2 < 0$ උදා: සෑම x සඳහාම $P(x)$ ඍණ වේ.
 $\lambda - 2 > 0$ නොවන නිසා $\Delta < 0$ උදා: සෑම x සඳහාම $P(x) > 0$ වේ.
 $\Delta = 9(\lambda + 2)^2 - 4(\lambda - 2)6 < 0$ උදා: සෑම x සඳහාම $P(x) > 0$ වේ.
 $\Delta < 0$ නොවන නිසා $\Delta = 9(\lambda^2 + 4\lambda + 4) - 24\lambda^2 + 48\lambda < 0$
 $\Rightarrow -15\lambda^2 + 84\lambda + 36 < 0$
 $\Rightarrow \lambda > 2$ නොවන නිසා $5\lambda^2 - 28\lambda - 12 > 0$
 $\Rightarrow \lambda > 2$ නොවන නිසා $(5\lambda + 2)(\lambda - 6) > 0$
 $\Rightarrow \lambda > 2$ නොවන නිසා $\lambda < -\frac{2}{5}$ නොවන නිසා $\lambda > 6$
 $\Rightarrow \lambda > 6$
 $\therefore x$ සෑම x සඳහාම $P(x) > 0$ වේ $\lambda > 6$ වන විට පමණි.

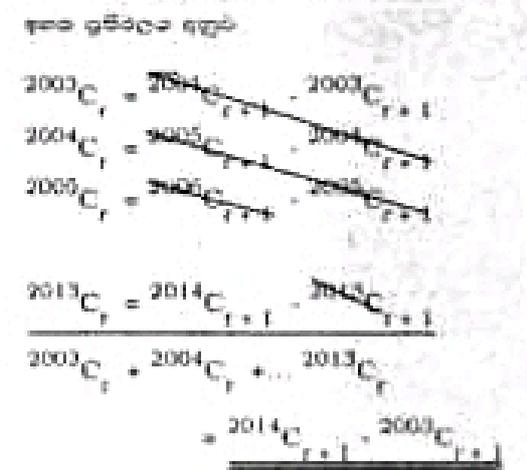
(ii) $P(x) = 0$ ග්‍රහණය කිරීම සඳහා $\Delta > 0$ විය යුතුය.
 $\Delta > 0$ නොවන නිසා $(5\lambda + 2)(\lambda - 6) < 0$
 $\Rightarrow \lambda > 2$ නොවන නිසා $-\frac{2}{5} < \lambda < 6$
 එසේ $\lambda \in (-\frac{2}{5}, 2) \cup (2, 6)$

(iii) $P(x) = 0$ ග්‍රහණය කිරීම සඳහා α හා β වන විට $\alpha > \beta$ විය යුතුය.
 $\alpha + \beta = \frac{3(\lambda + 2)}{\lambda - 2}$ නොවන නිසා $\alpha\beta = \frac{6\lambda}{\lambda - 2}$
 $\alpha - \beta = 1$ වේ.
 $(\alpha - \beta)^2 = (\alpha + \beta)^2 - 4\alpha\beta$
 $1 = \frac{9(\lambda + 2)^2}{(\lambda - 2)^2} - \frac{24\lambda}{\lambda - 2}$
 $\Rightarrow 9(\lambda - 2)^2 = 9(\lambda + 2)^2 - 24\lambda(\lambda - 2)$
 $\Rightarrow 9\lambda^2 - 36\lambda + 36 = 9\lambda^2 + 36\lambda + 36 - 24\lambda^2 + 48\lambda$
 $\Rightarrow 24\lambda^2 - 120\lambda = 0$
 $\Rightarrow \lambda^2 - 5\lambda = 0$
 $\Rightarrow \lambda(\lambda - 5) = 0$
 $\therefore \lambda = 0$ නොවන නිසා $\lambda = 5$

සංඛ්‍යාත්මක ලෙස	සංඛ්‍යාත්මක ලෙස
(a) 5, 1, 1, 1	$\frac{6!}{5!(1!)^3} = 56$
(b) 4, 2, 1, 1	$\frac{6!}{4!2!(1!)^2} = 420$
(c) 3, 3, 1, 1	$\frac{6!}{(3!)^2(1!)^2} = 280$
(d) 3, 2, 2, 1	$\frac{6!}{3!(2!)^2(1!)} = 840$
(e) 2, 2, 2, 2	$\frac{6!}{(2!)^4} = 105$

\therefore එකතුව වන්නේ $56 + 420 + 280 + 840 + 105 = 1701$

(iv) ${}^nC_{r+1} + {}^nC_r$
 $= \frac{n!}{(n-r-1)!(r+1)!} + \frac{n!}{(n-r)!(r)!}$
 $= \frac{n!(n-r)}{(n-r)!(r+1)!} + \frac{n!(r+1)}{(n-r)!(r+1)!}$
 $= \frac{n!(n-r+r+1)}{(n-r)!(r+1)!}$
 $= \frac{(n+1)!}{(n+1-r-1)!(r+1)!}$
 $= {}^{n+1}C_{r+1}$



(v) (a) $P(n) = 8(n+1)! > 2^{n+1}(n+2)$
 $n = 1$ උදා: L.H.S = $8 \times 2! = 16$, R.H.S = $4 \times 3 = 12$

а) Fill in the blank

Find the domain of the function $f(x) = \sqrt{x+2}$.

or $x+2 \geq 0 \Rightarrow x \geq -2$

or $x \in [-2, \infty)$

or $x \in \mathbb{R}, x \geq -2$

$$\begin{aligned} f(x) &= \sqrt{x+2} > 2^{n-1} \cdot (x+2) \\ &= \sqrt{x+2} > 2^{n-1} \cdot (x+2) \\ &= \sqrt{x+2} > 2^{n-1} \cdot (x+2) \\ &= \sqrt{x+2} > 2^{n-1} \cdot (x+2) \end{aligned}$$

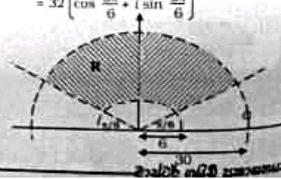
or $x+2 > 2^{n-1} \cdot (x+2)$

1. $x < 2$

$$(x+2) + (x-1) > 2x+1 > 5$$

$$\Rightarrow x > 2$$

or $x > 2$



$$|u| = 2 < 6$$

$$|u^2| = 4 < 6 \text{ cm } |u^3| = 32 > 30$$

or $|u^3| = 32 > 30$

$$\frac{d^2y}{dx^2} = \left(\frac{dy}{dx}\right) \sin x - \left(\frac{d^2y}{dx^2}\right) \cos x$$

$$-3 \left(\frac{d^2y}{dx^2}\right) \cos x - 3 \left(\frac{d^2y}{dx^2}\right) \sin x$$

$$\left(\frac{d^2y}{dx^2}\right) \cdot \left(\frac{d^2y}{dx^2}\right) \sin x + 2 \frac{dy}{dx} \cos x$$

$$\left(\frac{d^2y}{dx^2}\right) \sin x + 4 \left(\frac{d^2y}{dx^2}\right) \cos x$$

$$\left(\frac{d^2y}{dx^2}\right) - 5 \left(\frac{d^2y}{dx^2}\right) \sin x + 2 \frac{dy}{dx} \cos x$$

$$\left(\frac{d^2y}{dx^2}\right) = 0$$

$$\frac{d^2y}{dx^2} = 0$$

ա. խ) $\sqrt{x} = y$ առաջին օժիթ
 $x = y^2 \Rightarrow dx = 2y dy$

$$\int \frac{dx}{1+x^{1/3}}$$

$$= \int \frac{2y dy}{1+y^2}$$

$$= 3 \int_1^2 \left[\frac{1}{y} + \frac{1}{1+y} \right] dy$$

$$= 3 \left[\frac{1}{2} \ln y + \ln |1+y| \right]_1^2$$

$$= 3 \left[\ln 3 - \frac{1}{2} + \ln 2 \right]$$

$$= 3 \left[\frac{1}{2} + \ln \frac{3}{2} \right]$$

բ) $\int_0^1 x^2 e^{2x-3} dx$

$$= e^{-3} \int_0^1 x^2 e^{2x} dx$$

$$= e^{-3} \int_0^1 x^2 \frac{d}{dx} \left(\frac{e^{2x}}{2} \right) dx$$

$$= e^{-3} \left[\left(x^2 \frac{e^{2x}}{2} \right) - \int_0^1 \frac{e^{2x}}{2} 2x dx \right]$$

$$= e^{-3} \left[\frac{e^2}{2} - \int_0^1 x \frac{d}{dx} \left(\frac{e^{2x}}{2} \right) dx \right]$$

$$= e^{-3} \left[\frac{e^2}{2} - \left(x \frac{e^{2x}}{2} \right) + \int_0^1 \frac{e^{2x}}{2} dx \right]$$

$$= e^{-3} \left[\frac{e^2}{2} - \frac{e^2}{2} + \left(\frac{e^{2x}}{4} \right) \right]_0^1$$

$$= e^{-3} \left(\frac{e^2}{4} - \frac{1}{4} \right) = \frac{e^2}{4} (e^2 - 1)$$

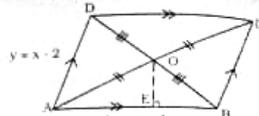
Կ) $\frac{1}{x(x^2+3)} = \frac{A}{x} + \frac{Bx+C}{x^2+3}$

$$\Rightarrow A = \frac{1}{3}, B = \frac{1}{3}, C = 0$$

$$\int \frac{dx}{x(x^2+3)} = \int \frac{dx}{3x} - \int \frac{x}{3(x^2+3)} dx$$

$$= \frac{1}{3} \left(\ln |x| - \frac{1}{2} \ln |x^2+3| \right) + D$$

$$= \frac{1}{3} \ln \frac{|x|}{\sqrt{x^2+3}} + D$$



ժ) $y = x - 2$
 $4y = x + 4$
 $\Rightarrow x = 4(x-2) = x+4$
 $\Rightarrow x = 4, y = 2$
 $A = (4, 2)$ օժիթ $C = (-4, -2)$
 BC թ ուղիղի $y = x + c$ օժիթ $(4, -2)$ կետը
 $-2 = 4 + c \Rightarrow c = -6$
 $\therefore BC$ թ ուղիղի $y = x - 6$

CD թ ուղիղի $4y = x + d$ օժիթ O
 C կետը $(-4, -2)$
 $-8 = -4 + d \Rightarrow d = -4$
 $4y = x - 4$

Կ) AC թ ուղիղի
 $\frac{y-0}{x-0} = \frac{-2-2}{-4-4} = \frac{-4}{-8} = \frac{1}{2} = 2y = x$

B կետը $(4, 2)$ օժիթ $4y = x + 4$ թ ուղիղի
 $4y - 4 = x + 4 - 4 \Rightarrow 4y - 4 = x$
 $4y - x = 4$
 $2y + x = 0$
 $4y - x = 4 \Rightarrow 2(2y - x) = 4 \Rightarrow 2y - x = 2$
 $2y + x = 0$
 $4y = x + 4$ Թ
 $BC \Rightarrow y = x + 2$ Թ
 7 օժիթ

$B = \left(\frac{4}{3}, \frac{2}{3} \right)$
 օժիթ AOB թ ուղիղի
 $= 4 \times \frac{1}{2} \times AB \times OE$
 $= 4 \times \frac{1}{2} \times \sqrt{\left(4 + \frac{4}{3}\right)^2 + \left(2 - \frac{2}{3}\right)^2} \times \frac{4}{\sqrt{17}}$

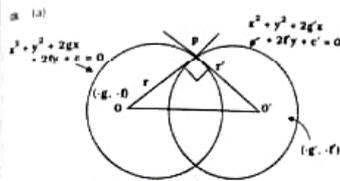
46

Թռչե՛ք ուճը ցրանում

$$= 2 \times \sqrt{\frac{256}{9} \times \frac{16}{9}} \times \frac{4}{\sqrt{17}}$$

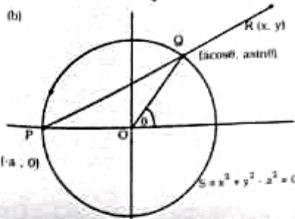
$$= 2 \times \frac{4}{\sqrt{17}} \times \frac{4\sqrt{17}}{3}$$

$$= \frac{32}{3} \text{ օժիթ}$$



Երկու շրջանների ուղիղ լիներանտ
 $|PO|^2 - (PO')^2 = |OO'|^2$
 օժիթ $(g-h)^2 + (f-k)^2 = (g+h)^2 + (f+k)^2$

օժիթ $r = \sqrt{g^2 + f^2 - c}$
 $r' = \sqrt{g'^2 + f'^2 - c'}$ թ
 $(g^2 + f^2 - c) = (g'^2 + f'^2 - c')$
 $= g^2 + f^2 - 2gg' + f^2 + f'^2 - 2ff'$
 $= -c - c' = -2gg' - 2ff'$
 $= 2gg' + 2ff' = c + c'$



$PO : OR = 1:1$ թ $R = (x, y)$
 $\Rightarrow \frac{-a+x}{2} = a \cos \theta$ թ $\frac{0+y}{2} = a \sin \theta$

47

$$\Rightarrow x = 2a \cos \theta + a$$

$$y = 2a \sin \theta$$

$$\Rightarrow R = (2a \cos \theta + a, 2a \sin \theta)$$

օժիթ $\cos \theta = \frac{x-a}{2a} \Rightarrow \sin \theta = \frac{y}{2a}$

$$\cos^2 \theta + \sin^2 \theta = 1$$

$$\Rightarrow \left(\frac{x-a}{2a} \right)^2 + \left(\frac{y}{2a} \right)^2 = 1$$

$$\Rightarrow (x-a)^2 + y^2 = 4a^2$$

օժիթ R , օժիթ $2a$ թ օժիթ $(a, 0)$ թ S կետը
 $\Rightarrow S$ թ ուղիղի $x^2 + y^2 - 2ax - 3a^2 = 0$
 S թ ուղիղի
 $S' = x^2 + y^2 + 2gx + 2fy + c = 0$ թ ուղիղի
 S թ S' շրջանները ուղիղ լիներանտ են
 $2g(0) + 2f(0) = c - a^2 \Rightarrow c = a^2$
 S թ S' շրջանները ուղիղ լիներանտ են
 $2g(a) + 2f(0) = c - 3a^2$
 $\Rightarrow -2ga = c - 3a^2$
 $\Rightarrow 2ga = c - 3a^2$
 $\Rightarrow 2ga = 2a^2 = g = a$
 $\therefore S' = x^2 + y^2 + 2ax + 2fy + a^2 = 0$
 S' թ y ուղիղի օժիթ $x = 0$ թ $\theta = 0$ թ $\theta = \pi$
 օժիթ $y^2 - 2fy + a^2 = 0$ թ $\theta = \frac{\pi}{2}$ թ $\theta = \frac{3\pi}{2}$
 օժիթ $4f^2 - 4a^2 = 0 \Rightarrow f^2 = a^2 \Rightarrow f = \pm a$
 օժիթ f թ S' ուղիղ լիներանտ ուղիղի S
 $S' = x^2 + y^2 + 2ax + 2ay + a^2 = 0$ թ
 $S' = x^2 + y^2 + 2ax + 2ay + a^2 = 0$ թ

Կ) (ա) $\theta = \sin \theta - \cos \theta$
 $\Rightarrow x^2 = \sin^2 \theta + \cos^2 \theta - 2 \sin \theta \cos \theta$
 $= 1 - \sin 2\theta$
 $\Rightarrow \sin 2\theta = 1 - x^2$

(բ) $y = \tan \theta + \cot \theta$
 $= \frac{\sin \theta}{\cos \theta} + \frac{\cos \theta}{\sin \theta}$
 $= \frac{\sin^2 \theta + \cos^2 \theta}{\sin \theta \cos \theta}$
 $= \frac{1}{\frac{1}{2} \sin 2\theta} = \frac{2}{\sin 2\theta}$
 $\Rightarrow \sin 2\theta = \frac{2}{y}$

$\Delta 1 - x^2 = \frac{2}{y} \Rightarrow y(1-x^2) - 2 = 0$

Թռչե՛ք ուճը ցրանում

$$\begin{aligned}
 (b) \quad & \sin 2x + \sin 4x + \sin 6x \\
 &= \sin 4x + (\sin 2x + \sin 6x) \\
 &= \sin 4x + 2\sin 4x \cos 2x \\
 &= (1 + 2\cos 2x) \sin 4x \\
 &= \sin x (\sin 2x + \sin 4x + \sin 6x) \\
 &= \sin x (1 + 2\cos 2x) \sin 4x \\
 &= \sin 4x (\sin x + 2\sin x \cos 2x) \\
 &= \sin 4x (\sin x + \sin 3x - \sin x) \\
 &= \sin 3x \sin 4x
 \end{aligned}$$

$x = \frac{\pi}{12}$ એ સરખાવો

$$\sin \frac{\pi}{12} \left(\frac{1}{2} + \frac{\sqrt{3}}{2} + 1 \right) = \frac{1}{\sqrt{2}} \times \frac{\sqrt{3}}{2}$$

$$\left(\frac{3 + \sqrt{3}}{2} \right) \sin \frac{\pi}{12} = \frac{\sqrt{3}}{2\sqrt{2}}$$

$$\sin \frac{\pi}{12} = \frac{2\sqrt{3}}{2\sqrt{2}(3 + \sqrt{3})}$$

$$\sin \frac{\pi}{12} = \frac{\sqrt{3}(3 - \sqrt{3})}{\sqrt{2}(9 - 3)}$$

$$= \frac{3(\sqrt{3} - 1)}{6\sqrt{2}}$$

$$= \frac{\sqrt{6} - \sqrt{2}}{4}$$

સરખાવો \therefore ABC ત્રિકોણમાં DO સરખાવવાથી

$$\frac{\sin A}{a} = \frac{\sin B}{b} = \frac{\sin C}{c} = k$$

$$\frac{\sin A}{a} = \frac{\sin B}{b} = \frac{\sin C}{c} = k$$

એ સરખાવો

$$a = b + \lambda c \text{ થી}$$

$$\Rightarrow \sin A = \sin B + \lambda \sin C$$

$$\begin{aligned}
 \Rightarrow \lambda \sin C &= \sin A - \sin B \\
 &= \sin(B + C) - \sin B \\
 &(\because A + B + C = \pi)
 \end{aligned}$$

$$\begin{aligned}
 \Rightarrow 2\lambda \sin \frac{C}{2} \cos \frac{C}{2} &= 2\sin \frac{C}{2} \cos \frac{(2B + C)}{2} \\
 &= 2\sin \frac{C}{2} \cos \frac{(2B + C)}{2}
 \end{aligned}$$

$$\Rightarrow \lambda \cos \frac{C}{2} = \cos \left(B + \frac{C}{2} \right)$$