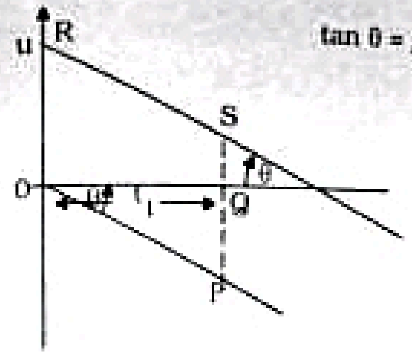
01. (a)



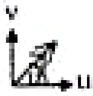
$\tan\theta = g$

ඇඳ ඇති ප්‍රවේග කාල වක්‍ර අනුව A අංශුව සිරස්ව ගමන් කළ විස්ථාපනය = වර්ගඵලය (OPQ Δ) හා B අංශුව සිරස්ව ගමන් කළ විස්ථාපනය = වර්ගඵලය (ORSQ ⏢) එකතුව h ට සමාන විය යුතුය. එම මුලු කාලය $t = t_1$ (OQ) යැයි ගනිමු.

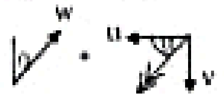
ORSQ ⏢ + OPQ Δ = ORSP ▱

ORSP ▱ = OR × OQ

$\therefore h = u \times t_1 \Rightarrow t_1 = \dfrac{h}{u}$

(b) (කු, පො) =  , (බෝ, පො) = w

(බෝ, කු) = (බෝ, පො) + (පො, කු)

= 

= 

(බෝ, කු) = → $(w\sin\theta - u)$ + ↑ $(w\cos\theta - v)$

බෝට්ටුවට කෙටිම කාලයකින් ගමන් කිරීමට සාපේක්ෂ ප්‍රවේගය ↑ විය යුතුය. එම නිසා → ප්‍රවේගය, $w\sin\theta - u = 0$ විය යුතුය. එනම්

$\sin\theta = \dfrac{u}{w}$; $\theta < \dfrac{\pi}{2}$ නිසා

$\sin\theta < 1 \Rightarrow \dfrac{u}{w} < 1 \Rightarrow u < w$

ගැටීමට ගතවන කාලය = $\dfrac{d}{w\cos\theta - v}$

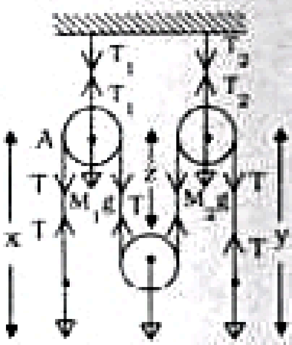
$= \dfrac{d}{w\sqrt{1-\sin^2\theta} - v} = \dfrac{d}{\sqrt{w^2-u^2} - v}$

02. $x + y + 2z$ = නියතයකි
කාලය t විෂයෙන් අවකලනයෙන්

$\ddot{x} + \ddot{y} + 2\ddot{z} = 0$ වේ.

$\downarrow f_1 = \ddot{x}$ ද, $\downarrow f_2 = \ddot{y}$ ද

එවිට ගණිත $\ddot{z} = \uparrow \dfrac{(f_1 + f_2)}{2}$



(a) $(m_1, E) = \downarrow f_1$

$(m_2, E) = \downarrow f_2$

$(M_3, E) = \uparrow \dfrac{f_1 + f_2}{2}$

$\underline{F} = m\underline{a}$ යෙදීමෙන්

$\downarrow m_1 g - T = m_1 f_1$ ——— ①

$\downarrow m_2 g - T = m_2 f_2$ ——— ②

$\uparrow 2T - M_3 g = M_3 \dfrac{(f_1 + f_2)}{2}$ ——— ③

① න් $f_1 = g - \dfrac{T}{m_1}$

② න් $f_2 = g - \dfrac{T}{m_2}$

③ ට ආදේශයෙන්

$2T - M_3 g = \dfrac{M_3}{2}\left[g - \dfrac{T}{m_1} + g - \dfrac{T}{m_2}\right]$

$\dfrac{4T}{M_3} - 2g = 2g - \dfrac{T}{m_1} - \dfrac{T}{m_2} \quad \left(\div \dfrac{M_3}{2} \text{ න්}\right)$

$T\left[\dfrac{4}{m_3} + \dfrac{1}{m_1} + \dfrac{1}{m_2}\right] = 4g$

$T\left[\dfrac{4m_1m_2 + M_3(m_1+m_2)}{M_3 m_1 m_2}\right] = 4g$

$T = \dfrac{4m_1m_2M_3 g}{4m_1m_2 + M_3(m_1+m_2)}$

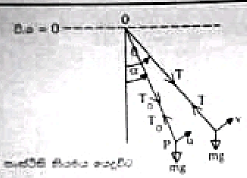
$T_1 = 2T + M_1 g$; $T_2 = 2T + M_2 g$

සිවිලිම මගින් ලබා දෙන බලය

$T_1 + T_2 = 4T + g(M_1 + M_2)$

$= \dfrac{16 m_1 m_2 M_3 g}{4m_1m_2 + M_3(m_1+m_2)} + g(M_1 + M_2)$

(b)



ශක්ති සංස්ථිති නියමය යෙදීමෙන්

$\frac{1}{2}mu^2 - mga\cos\alpha$

$= \frac{1}{2}mv^2 - mga\cos\theta$ —— ①

$\underline{F} = m\underline{a}$ යෙදීමෙන්

$T - mg\cos\theta = m\frac{v^2}{a}$ —— ②

① න් $v^2 = u^2 - 2ga\cos\alpha + 2ga\cos\theta$ —— ③

② හි ආදේශයෙන්

$T = mg\cos\theta + \frac{m}{a}[u^2 - 2ga\cos\alpha + 2ga\cos\theta]$

$T = \frac{m}{a}[u^2 - 2ga\cos\alpha + 3ga\cos\theta]$ —— ④

③ හා ④ අනුව θ වැඩිවන විට v හා T දෙකම අඩුවේ [illegible]

$\frac{\pi}{2} < \theta < \pi$ විට $v^2 > 0$ හා $T = 0$ [illegible]

$T = 0$ වන්නේ $\theta = \theta_1$ විටයි.

$\cos\theta_1 = \frac{2ga\cos\alpha - u^2}{3ga}$ වේ

එවිට $\theta_1 = \cos^{-1}\left[\frac{1}{3}\left(2\cos\alpha - \frac{u^2}{ga}\right)\right]$ වේ

[illegible] OP [illegible] $\cos^{-1}\left[\frac{1}{3}\left(2\cos\alpha - \frac{u^2}{ga}\right)\right]$ [illegible]

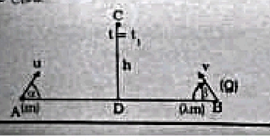
[illegible]

$\frac{\pi}{2} < \theta_1 < \pi$ විට $-1 < \cos\theta_1 < 0$ බැවින්

[illegible] $-1 < \frac{2ga\cos\alpha - u^2}{3ga} < 0$ විය යුතුය.

මෙමගින් $ga(3 + 2\cos\alpha) > u^2 > 2ga\cos\alpha$ බව ලැබේ.

03.



$t = t_1$ විට [illegible] AB ට h [illegible] C [illegible]

$S = ut + \frac{1}{2}at^2$ ↑ යෙදීමෙන්

Ⓟ ට $h = u\sin\alpha\, t_1 - \frac{1}{2}gt_1^2$ —— ①

Ⓠ ට $h = v\sin\beta\, t_1 - \frac{1}{2}gt_1^2$ —— ②

① = ② න් $u\sin\alpha = v\sin\beta$ බව ලැබේ ($\because t_1 \neq 0$ නිසා)

$v = u + at$ ↑ [illegible] T ($T \le t_1$) [illegible]

Ⓟ ට ↑ $v_P = u\sin\alpha - gT$ —— ③

Ⓠ ට ↑ $v_Q = v\sin\beta - gT$ —— ④

$u\sin\alpha = v\sin\beta$ නිසා $v_P = v_Q$ වේ. [illegible] $T \le t_1$ [illegible]

[illegible]

∴ P හා Q [illegible]

(i) $T = t_1$ විට ③ ට අනුව $v_P = 0$ නිසා (C [illegible])

$0 = u\sin\alpha - gt_1 \Rightarrow t_1 = \frac{u\sin\alpha}{g}$ —— ⑤

A → C [illegible]

$AD = u\cos\alpha \cdot t_1$

B → C [illegible]

$DB = v\cos\beta \cdot t_1$

$AD + DB = AB = \frac{u^2\sin 2\alpha}{g} = \frac{2u^2\sin\alpha\cos\alpha}{g}$

$= \frac{2u\sin\alpha}{g} \cdot u\cos\alpha$

$= 2t_1 u\cos\alpha$ (⑤ න්)

$u\cos\alpha\, t_1 + v\cos\beta\, t_1 = 2t_1 \cdot u\cos\alpha$

$\underline{\underline{v\cos\beta = u\cos\alpha}}$

(ii) $\xrightarrow{u\cos\alpha}$ (m) (λm) $\xleftarrow{v\cos\beta = u\cos\alpha}$ ([illegible])

(m(1 + λ)) → v_0 ([illegible])

ගම්‍යතා සංස්ථිති නියමයට අනුව

→ $mu\cos\alpha - \lambda m \cdot u\cos\alpha = m(1 + \lambda)v_0$

$\underline{\underline{u\cos\alpha\frac{(1 - \lambda)}{(1 + \lambda)} = v_0}}$

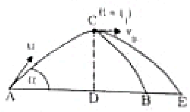
(iii) P [illegible] CD = h නම්.

↑ $v^2 = u^2 + 2as$ යෙදීමෙන්

$0 = u^2\sin^2\alpha - 2gh$

$h = \frac{u^2\sin^2\alpha}{2g}$

[illegible] E හිදී [illegible]



C → E [illegible]

↓ $S = ut + \frac{1}{2}at^2$ යෙදීමෙන්

$\frac{u^2\sin^2\alpha}{2g} = 0 + \frac{1}{2}gt_2^2$

$t_2 = \frac{u\sin\alpha}{g} = t_1$ වේ

[illegible] P [illegible]

$AD = u\cos\alpha \times t_1 = u\cos\alpha \cdot \frac{u\sin\alpha}{g}$

$DE = v_0 \times t_2$ ([illegible])

$AE = AD + DE$ බැවින්

$\therefore AE = \frac{u^2\sin\alpha\cos\alpha}{g} + \frac{(1 - \lambda)}{(1 + \lambda)}u\cos\alpha \cdot \frac{u\sin\alpha}{g}$

$= \frac{u^2\sin\alpha\cos\alpha}{g}\left[1 + \frac{(1 - \lambda)}{(1 + \lambda)}\right]$

$\underline{\underline{AE = \frac{u^2\sin 2\alpha}{g(1 + \lambda)}}}$

P ට සාපේක්ෂව Q හි [illegible] නම් $\lambda \to 0$ වේ

එවිට $AE = \frac{u^2\sin 2\alpha}{g}$ එනම් E = B වේ

Q ට සාපේක්ෂව P හි [illegible] නම් $\lambda \to \infty$ වේ

එවිට AE = 0, එනම් E = A වේ

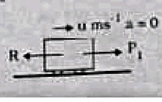
P හා Q හි [illegible] නම් λ = 1

එවිට $AE = \frac{u^2\sin 2\alpha}{2g} = \frac{AB}{2}$

එවිට E යනු AB හි මධ්‍ය ලක්ෂ්‍යයයි.

$\because AB = \frac{u^2\sin 2\alpha}{g}$ බැවිනි.

04. (a)

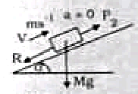


[illegible] $P_1 = \frac{1000 \times H}{u}$

$\underline{F} = m\underline{a}$ යෙදීමෙන්

→ $P_1 - R = M \times 0$

$R = \frac{1000 \times H}{u}$ N



$P_2 = \frac{1000 \times H}{v}$

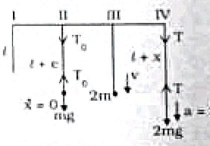
↗ $P_2 - R - Mg\sin\alpha = 0$

$\frac{1000H}{v} - \frac{1000H}{u} = Mg\sin\alpha$

$1000H\left(\frac{u - v}{vu}\right) = Mg\sin\alpha$

$\underline{\underline{H = \frac{uv\,Mg\sin\alpha}{1000(u - v)}\ kW}}$

(b)



II අවස්ථාව, λ යනු [illegible] නම්.

$T_0 = mg$ ($\because \ddot{x} = 0$ නිසා)

$T_0 = \frac{\lambda c}{l}$

$\frac{\lambda c}{l} = mg$

$\underline{\underline{\lambda = \frac{lmg}{c}}}$

III අවස්ථාව, ගම්‍යතා සංස්ථිති නියමය

↓ $m \times 0 + m \times u = 2m \times v$

u යනු Q [illegible] c [illegible]

$u^2 = 0 + 2gc$

$v = \frac{u}{2} = \sqrt{\frac{gc}{2}}$

IV අවස්ථාව,

$T = \frac{\lambda x}{l} = \frac{lmg}{c} \times \frac{x}{l} = \frac{mg\,x}{c}$

$\underline{F} = m\underline{a}$ ↓ යෙදීමෙන්

$2mg - T = 2m\ddot{x}$

$2mg - \frac{mgx}{c} = 2m\ddot{x}$

$\ddot{x} = \frac{-g}{2c}x + g$

$\ddot{x} + \omega^2 x - 2\omega^2 c = 0 \quad (\because \omega^2 = \frac{g}{2c})$

$\ddot{x} + \omega^2 (x - 2c) = 0$

$x = 2c + a\cos\omega t + b\sin\omega t$ ——— ①

$t = 0$ විට $x = c$ බැවින්

$c = 2c + a \Rightarrow \underline{a = -c}$

①, t විෂයෙන් අවකලනයෙන්

$\dot{x} = -a\omega\sin\omega t + b\omega\cos\omega t$ ——— ②

$t = 0$ විට $\dot{x} = \frac{u}{2} = \sqrt{\frac{gc}{2}}$ නිසා

$\sqrt{\frac{gc}{2}} = b\omega = b\sqrt{\frac{g}{2c}} \Rightarrow \underline{b = c}$

② න් $\dot{x} = c\omega\sin\omega t + c\omega\cos\omega t$

$\dot{x} = c\omega(\sin\omega t + \cos\omega t)$

$t = t_1$ විට $\dot{x} = 0$ වේ නම්

$0 = c\omega(\sin\omega t_1 + \cos\omega t_1)$

$\sin\omega t_1 = -\cos\omega t_1$

$\tan\omega t_1 = -1$

$\omega t_1 = \frac{3\pi}{4}$

$t_1 = \frac{3\pi}{4\omega} = \frac{3\pi}{4} \times \frac{1}{\sqrt{\frac{2c}{g}}} = \frac{3\pi}{4}\sqrt{\frac{2c}{g}}$

① න් $x = 2c - c\cos\left(\omega \cdot \frac{3\pi}{4\omega}\right) + c\sin\left(\omega \cdot \frac{3\pi}{4\omega}\right)$

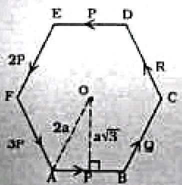
$x = 2c - c\cos\frac{3\pi}{4} + c\sin\frac{3\pi}{4}$

$= 2c - c\left(-\frac{1}{\sqrt{2}}\right) + c\left(\frac{1}{\sqrt{2}}\right)$

$= 2c + \sqrt{2}\,c$

$= \sqrt{2}\,c(1 + \sqrt{2}) \quad (\dot{x} = 0$ වන$)$

05. (a) (i)



$\rightarrow P + Q\cos 60 - R\cos 60 - P - 2P\cos 60 + 3P\cos 60 = 0$

$R - Q = P$ ——— ①

$\uparrow (Q + R - 2P - 3P)\cos 30 = 0$

$Q + R = 5P$ ——— ②

① + ② $\underline{R = 3P\,N}$; ② - ① $\underline{Q = 2P\,N}$

$(P + Q + R + P + 2P + 3P)\,a\sqrt{3}$

$= (7P + 3P + 2P)\,a\sqrt{3}$

$= \underline{12\sqrt{3}\,a\,P\,N\,m}$

(ii) AD

$P\cos 30 - 3P\cos 30 - P\cos 30 + R\cos 30 = 0$

$\underline{R = 3P\,N}$

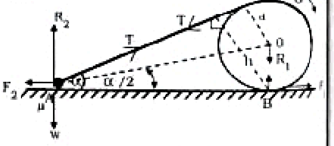
O

$(P + Q + R + P + 2P + 3P)\,a\sqrt{3} = 0$

$7P + Q + 3P = 0$

$\underline{Q = -10\,PN}$

(b)



$\rightarrow T\cos\alpha = F_2$ ——— ①

$\uparrow T\sin\alpha + R_2 = w$ ——— ②

$F_2 = \mu R_2$ ——— ③

① හා ③ $T\cos\alpha = \mu R_2$ ——— ④

② හා ④ න් $T\cos\alpha = \mu(w - T\sin\alpha)$

$T(\cos\alpha + \mu\sin\alpha) = \mu w$

$T = \frac{\mu w}{\cos\alpha + \mu\sin\alpha}$

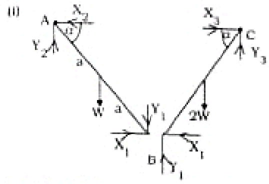
B වටා සූර්ණ ගැනීමෙන්

$T \times AB\sin\alpha = G \quad ; AB = a\cot\frac{\alpha}{2}$

$T \times a\cot\frac{\alpha}{2}\sin\alpha = G$

$\frac{\mu wa}{(\cos\alpha + \mu\sin\alpha)}(1 + \cos\alpha) = G$

06. (a) (i)



A වටා AB ට

$W \times a\cos\alpha + Y_1 \times 2a\cos\alpha - X_1 \times 2a\sin\alpha = 0$ ——— ①

C වටා CB ට

$2W \times a\cos\alpha - Y_1 \times 2a\cos\alpha - X_1 \times 2a\sin\alpha = 0$ ——— ②

① + ② න්

$3W\cos\alpha = 4X_1\sin\alpha$

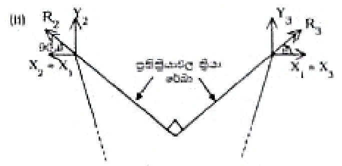
$X_1 = \frac{3W\cot\alpha}{4}$

① - ② න්

$-W\cos\alpha + 4Y_1\cos\alpha = 0$

$Y_1 = \frac{W}{4}$

(ii)



$\tan(90 - \theta) = \frac{Y_2}{X_2} \; ; \; \tan\theta = \frac{Y_3}{X_3}$

$\cot\theta = \frac{Y_2}{X_2} \quad \tan\theta \cdot \cot\theta = 1$

$\frac{Y_3}{X_3} \cdot \frac{Y_2}{X_2} = 1$

$\frac{Y_2}{X_2} = \frac{X_3}{Y_3}$ වේ $\Rightarrow Y_2Y_3 = X_2X_3$ ——— ③

AB

$\rightarrow X_1 = X_2 = \frac{3W\cot\alpha}{4}$

$\uparrow Y_2 = W + Y_1 = \frac{5W}{4}$

BC

$\rightarrow X_3 = X_1 = \frac{3W\cot\alpha}{4} = X_2$

$\uparrow Y_3 = 2W - Y_1 = \frac{7W}{4}$

③

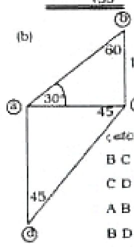
$\frac{5W}{4} \times \frac{7W}{4} = \frac{3W\cot\alpha}{4} \times \frac{3W\cot\alpha}{4}$

$35 = 9\cot^2\alpha$

$\tan^2\alpha = \frac{9}{35}$

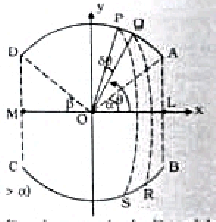
$\tan = \frac{3}{\sqrt{35}}$

(b)



B C		$1000\sqrt{3}$ N
C D		2000 N
A B		$1000\sqrt{6}$ N
B D		$1000\sqrt{3}$ N

07.



X

$P\hat{O}Q = \delta\theta$ ද $Q\hat{O}X = \theta$ ද

PQ $= a\,\delta\theta$

$= a\sin\theta$

$= (2\pi a\sin\theta)(a\,\delta\theta)\,\sigma$

y $= a\cos\theta$

[illegible] ස්කන්ධ කේන්ද්‍රය y අක්ෂයේ සිට $\bar{x}$ දුරකින් [illegible]

$$\bar{x} = \frac{\int m_r x_r}{\int m_r}$$ [illegible]

$$\bar{x} = \frac{\int_{\alpha}^{\pi-\beta} 2\pi a \sin\theta \cdot a d\theta\, \sigma \cdot a\cos\theta}{\int_{\alpha}^{\pi-\beta} 2\pi a \sin\theta \cdot a d\theta\, \sigma}$$

$$= \frac{\pi a^3 \sigma \int_{\alpha}^{\pi-\beta} 2\sin\theta\cos\theta\, d\theta}{2\pi a^2 \sigma \int_{\alpha}^{\pi-\beta} \sin\theta\, d\theta}$$

$$= \frac{\pi a^3 \sigma [\sin^2\theta]_{\alpha}^{\pi-\beta}}{-2\pi a^2 \sigma [\cos\theta]_{\alpha}^{\pi-\beta}}$$

$$= \frac{\pi a^3 \sigma [\sin^2\beta - \sin^2\alpha]}{-2\pi a^2 \sigma [-\cos\beta - \cos\alpha]}$$

[$\because \sin(\pi-\beta) = \sin\theta$ හා $\cos(\pi-\beta) = -\cos\beta$ නිසා]

$$= \frac{\pi a^3 \sigma [1 - \cos^2\beta - 1 + \cos^2\alpha]}{2\pi a^2 \sigma [\cos\alpha + \cos\beta]}$$

$$\bar{x} = \frac{\pi a^3 \sigma [\cos\alpha - \cos\beta](\cos\alpha + \cos\beta)}{2\pi a^2 \sigma [\cos\alpha + \cos\beta]} \quad \text{—— ①}$$

(i) [illegible] $\int m_r$ [illegible] ස්කන්ධය $= 2\pi a^2 \sigma (\cos\alpha + \cos\beta)$ [illegible]

ස්කන්ධ කේන්ද්‍රය O සිට $\frac{a}{2}(\cos\alpha - \cos\beta)$ දුරකින් [illegible] (① [illegible])

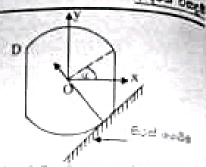
(ii) ML [illegible] $= a\cos\alpha + a\cos\beta$

ML හි [illegible] O සිට ඇති දුර

$$= a\cos\alpha - \frac{(a\cos\alpha + a\cos\beta)}{2}$$

$$= \frac{(a\cos\alpha - a\cos\beta)}{2} = \bar{x} \text{ වේ}$$

වස්තුව	ස්කන්ධය	ස.කේ.ට y අක්ෂයේ සිට දුර
[illegible]	$2\pi a^2\sigma(\cos\alpha + \cos\beta)$	$\frac{a}{2}(\cos\alpha - \cos\beta)$
[illegible]	$\pi a^2 \sin^2\beta\sigma$	$-a\cos\beta$
[illegible]	$2\pi a^2\sigma(\cos\alpha + \cos\beta) + \pi a^2 \sin^2\beta\sigma$	$\bar{x}$



[illegible] O [illegible] O [illegible]

එනම් $\bar{x} = 0$, y [illegible]

$$\frac{a}{2}(\cos\alpha + \cos\beta) \cdot \frac{a}{2}(\cos\alpha - \cos\beta) - \pi a^2 \sin^2\beta\, \sigma\, a\cos\beta = 0$$

$$\cos^2\alpha - \cos^2\beta - \sin^2\beta\cos\beta = 0$$

$$1 - \sin^2\alpha - 1 + \sin^2\beta - \sin^2\beta\cos\beta = 0$$

$$\sin^2\beta(1 - \cos\beta) = \sin^2\alpha$$

$$\therefore \sin\alpha = \sin\beta\sqrt{1 - \cos\beta}$$

08. (a) A : [illegible]
B : [illegible]
$P(A) = p$ හා $P(B/A) = q$ [illegible]

$$P(B/A) = \frac{P(A \cap B)}{P(A)}$$

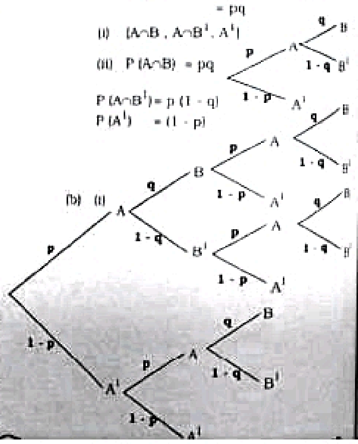
$$P(A \cap B) = P(A)\,P(B/A) = pq$$

(i) $\{A \cap B,\ A \cap B^1,\ A^1\}$

(ii) $P(A \cap B) = pq$

$P(A \cap B^1) = p(1 - q)$

$P(A^1) = (1 - p)$



(b) (i)

(ii) [illegible]
$$= P(A \cap B \cap A \cap B) = (pq)(pq) = p^2q^2$$

(iii) X : [illegible], Y : [illegible]

[illegible]

$$(X \cap Y) = (A \cap B \cap A \cap B^1) \cup (A \cap B^1 \cap A \cap B)$$

[illegible]

$$P(X \cap Y) = pq\,p(1-q) + p(1-q)\,pq$$

$$P(X \cap Y) = 2p^2q(1-q)$$

$$X = (A \cap B \cap A \cap B) \cup (A \cap B \cap A \cap B^1) \cup (A \cap B^1 \cap A \cap B) \cup (A \cap B^1 \cap A \cap B^1)$$

$$P(X) = pqpq + pqp(1-q) + pqp(1-q) + p(1-q)p(1-q)$$

$$= pqp(q + 1 - q + 1 - q) + p^2(1-q)^2$$

$$= pqp(2 - q) + p^2(1-q)^2$$

$$= p^2(2q - q^2 + 1 - 2q + q^2)$$

$$= p^2$$

[illegible] $P(Y \mid X)$ වේ.

$$P(Y \mid X) = \frac{P(Y \cap X)}{P(X)} = \frac{2p^2q(1-q)}{p^2} = 2q(1-q)$$

(iv) [illegible] $= C$
[illegible] $= D$

$$C = (A \cap B \cap A^1) \cup (A \cap B^1 \cap A^1) \cup (A^1 \cap A \cap B) \cup (A^1 \cap A \cap B^1)$$

$$P(C) = pq(1-p) + p(1-q)(1-p) + (1-p)pq + (1-p)p(1-q)$$

$$= p(1-p)(q + 1 - q + q + 1 - q)$$

$$= 2p(1-p)$$

$$C \cap D = (A \cap B \cap A^1) \cup (A^1 \cap A \cap B)$$

$$P(C \cap D) = pq(1-p) + (1-p)pq = 2pq(1-p)$$

$$\therefore P(D \mid C) = \frac{P(C \cap D)}{P(C)} = \frac{2pq(1-p)}{2p(1-p)}$$

$$= q$$

09. $x_1, x_2, \ldots, x_n$ [illegible] n [illegible]

$$\bar{x} = \frac{x_1 + x_2 + \cdots + x_n}{n} = \frac{1}{n}\sum_{i=1}^{n} x_i$$

[illegible] $= \frac{1}{n}\sum_{i=1}^{n}(x_i - \bar{x})^2$ [illegible]

(i) $\bar{x} = \frac{1}{n}\sum_{i=1}^{n} x_i$ —— ① හා

$\bar{y} = \frac{1}{m}\sum_{i=1}^{m} y_i$ —— ② වේ.

$$\bar{z} = \frac{x_1 + x_2 + \cdots + x_n + y_1 + y_2 + \cdots + y_m}{m + n}$$

① න් $\sum_{i=1}^{n} x_i = n\bar{x}$ ද

② න් $\sum_{i=1}^{m} y_i = m\bar{y}$ ද [illegible]

$$\bar{z} = \frac{n\bar{x} + m\bar{y}}{m + n}$$

(ii) $\sum_{i=1}^{n}(x_i - \bar{z})^2$

$$= \sum_{i=1}^{n}(x_i - \bar{x} + \bar{x} + \bar{z})^2$$

$$= \sum_{i=1}^{n}(x_i - \bar{x})^2 + 2\sum_{i=1}^{n}(x_i - \bar{x})(\bar{x} - \bar{z}) + \sum_{i=1}^{n}(\bar{x} - \bar{z})^2$$

$$= n\sigma_1^2 + \sigma + \sum_{i=1}^{n} d_1^2 \quad (\because \bar{x} - \bar{z} = d_1 \text{ [illegible]})$$

$$= n\sigma_1^2 + nd_1^2 \quad (\because \sum_{i=1}^{n}(x_i - \bar{x}) = 0 \text{ [illegible]})$$

(iii) $$\sigma^2 = \frac{1}{n+m}\left\{(x_1 - \bar{z})^2 + (x_2 - \bar{z})^2 + \cdots + (x_n - \bar{z})^2 + (y_1 - \bar{z})^2 + (y_2 - \bar{z})^2 + \cdots + (y_m - \bar{z})^2\right\}$$

$$= \frac{1}{n+m}\left\{\sum_{i=1}^{n}(x_i - \bar{z})^2 + \sum_{i=1}^{m}(y_i - \bar{z})^2\right\}$$

$$= \frac{1}{n+m}\left\{n(\sigma_1^2 + d_1^2) + m(\sigma_2^2 + d_2^2)\right\}$$

(II [illegible])

(b)

පන්තිය	මධ්‍ය අගය	අපගමනය (d_i)	u_i	f_i	f_iu_i	$f_iu_i^2$
30 - 34	32	-15	-3	5	-15	45
35 - 39	37	-10	-2	10	-20	40
40 - 44	42	-5	-1	15	-15	15
45 - 19	47	0	0	30	0	0
50 - 54	52	+5	1	5	+5	5
55 - 59	57	+10	2	5	+10	20
				70	-35	125

(i) මධ්‍යන්‍යය $\overline{x} = A + C\left(\frac{\Sigma f_i u_i}{\Sigma f_i}\right)$

$$= 47 + \frac{5(-35)}{70} = 44.5$$

විචලතාව $\sigma_1^2 = C^2\left[\frac{\Sigma f_i u_i^2}{\Sigma f_i} - \left(\frac{\Sigma f_i u_i}{\Sigma f_i}\right)^2\right]$

$$= 25\left[\frac{125}{70} - \left(\frac{-35}{70}\right)^2\right]$$

$$\overline{x} = 25\left[\frac{25}{14} - \left(\frac{1}{2}\right)^2\right] = \frac{25 \times 43}{28}$$

$$= 38.39$$

(ii) (a) I කොටසට අදාළ

$$38 = \frac{(70 \times 44.5) + (30 \times \overline{y})}{70 + 30}$$

$(\because m = 70,\ n = 30\ \overline{x} = 44.5$ නිසා)

$\overline{y} = 22.83$

(a) III කොටසට අදාළ

$$\sigma^2 = \frac{1}{m + n}\left(m(\sigma_2^2 + d_2^2) + n(\sigma_1^2 + d_1^2)\right)$$

$m = 30,\ n = 70,\ \sigma_1^2 = 38.39,$

$d_2 = \overline{y} - \overline{z} = 22.83 - 38 = -15.17$

$d_1 = \overline{x} - \overline{z} = 44.5 - 38 = 6.5;\ \sigma^2 = 144$

$$144 = \frac{1}{30 + 70}\left(30\left(\sigma_2^2 + (-15.17)^2\right) + 70\left(38.39 + (6.5)^2\right)\right)$$

$$14400 = 30\sigma_2^2 + 6903.9 + 2687.3 + 2957.5$$

$$1851.3 = 30\sigma_2^2$$

$$\underline{\sigma_2^2 = 61.71}$$

*** ***