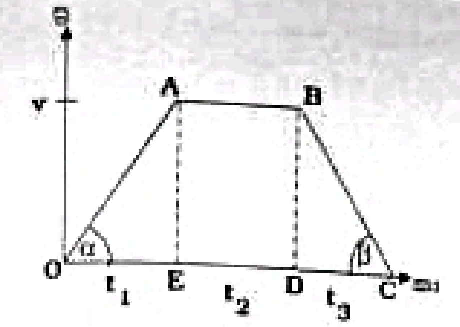
01.



$\tan\alpha = \frac{1}{3}$ $\qquad$ $\tan\beta = 1$

(a) [illegible]
එවිට $t_1 + t_2 + t_3 = 60$ —— ① හා

OAE Δ + ABDE ▭ + BDC Δ = 432 —— ②

OAE Δ න්

$\tan\alpha = \frac{1}{3} = \frac{v}{t_1} \Rightarrow t_1 = 3v$ —— ③

BDC Δ

$\tan\beta = 1 = \frac{v}{t_3} \Rightarrow t_3 = v$ —— ④

$\left(\frac{1}{2} \times t_1 \times v\right) + (t_2 \times v) + \left(\frac{1}{2} \times t_3 \times v\right) = 432$

$\left(\frac{1}{2} \times 3v \times v\right) + (t_2 v) + \left(\frac{1}{2} \times v \times v\right) = 432$ —— ⑤

①, ③, ④ න් $3v + t_2 + v = 60$

$\Rightarrow t_2 = 60 - 4v$ —— ⑥

⑤ හා ⑥ න්

$\frac{3v^2}{2} + (60 - 4v)v + \frac{1}{2}v^2 = 432$

$\Rightarrow v^2 - 30v + 216 = 0$

$(v - 12)(v - 18) = 0$

$v = 12$ හෝ $v = 18$

[illegible] $v = 18$ [illegible]

(∵ එවිට $t_2 < 0$ නිසා)

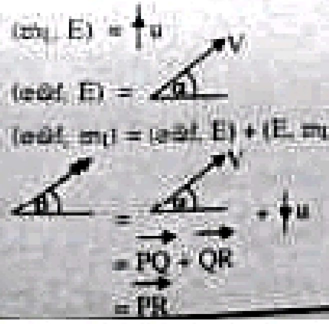
$v = 12$ විට $t_1 = 36, t_2 = 12, t_3 = 12$ වේ.

OAE Δ : ABDE ▭ : BDC Δ

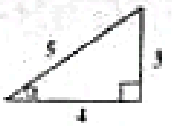
$= \left(\frac{1}{2} \times 3 \times 12 \times 12\right) : (12 \times 12) : \left(\frac{1}{2} \times 12 \times 12\right)$

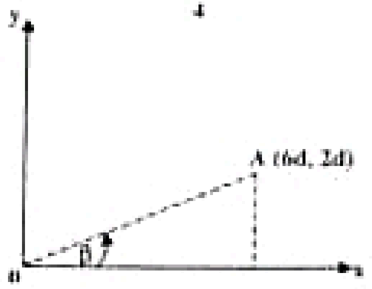
$= \frac{3}{2} : 1 : \frac{1}{2} = 3 : 2 : 1$

(b) (ගු, E) = $\uparrow u$

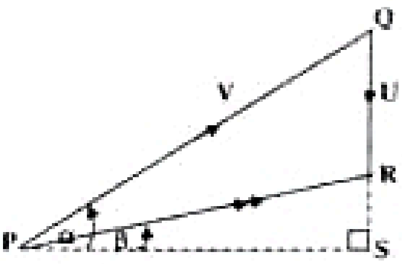


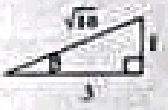
එවිට $\tan\alpha = \frac{3}{4}$ [illegible]





$\tan\beta = \frac{2d}{6d} = \frac{1}{3}$





(∵ α > β නිසා)

PQS Δ න්

$QS = V\sin\alpha = \frac{3V}{5} \Rightarrow RS = QS - QR = \frac{3V}{5} - u$

$PS = V\cos\alpha = \frac{4V}{5}$

PRS Δ න්

$\tan\beta = \frac{RS}{PS} \Rightarrow \frac{1}{3} = \frac{\frac{3V}{5} - u}{\frac{4V}{5}} = \frac{3V - 5u}{4V} \Rightarrow 4V = 9V - 15u$

$\Rightarrow 5V = 15U \Rightarrow V = \underline{\underline{3u}}$

[illegible] $= \frac{\sqrt{(6d)^2 + (2d)^2}}{|\overrightarrow{PR}|}$ —— ① වේ.

[illegible] $\frac{6d}{|\overrightarrow{PS}|}$ [illegible]

$t = \frac{6d}{\frac{4V}{5}} = \frac{30d}{4v} = \frac{30d}{4 \times 3u} = \frac{30d}{12u} = \frac{5d}{2u}$ වේ.

① [illegible]

02. (a) (ලෙස්, E) $= \uparrow F$

(km ලෙස්) $= \downarrow f$ } [illegible]

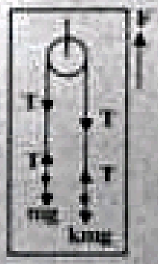
(m ලෙස්) $= \uparrow f$

(km E) = (km ලෙස්) + (ලෙස් E)

$= \downarrow f + \uparrow F = \downarrow (f - F)$

(m E) = (m ලෙස්) + (ලෙස් E)

$= \uparrow f + \uparrow F = \uparrow (f + F)$



E = mg යොදමින්

(km) ↑ $kmg - T = km(f - F)$ — ①

(m) ↑ $T - mg = m(f + F)$ — ②

① + ② න් $mg(k-1) = mf(k+1) - mF(k-1)$

$g(k-1) + F(k-1) = f(k+1)$

$$\frac{(k-1)(g+F)}{(k+1)} = f$$

② × k - ① න්

$T + kT - 2kmg = 2kmF$

$T(1+k) = 2km(F+g)$

$$T = 2km\frac{(F+g)}{(1+k)}$$

[illegible] (km) [illegible]

$(km\,k) = \frac{1}{?}(f - F) = 0$ [illegible]

∴ [illegible] $k\,mg = T$ [illegible] (① න්)

$$\Rightarrow \quad k\,mg = 2km\frac{(F+g)}{(1+k)} \Rightarrow g + gk = 2F + 2g$$

$$\Rightarrow \quad gk - g = 2F \Rightarrow \frac{g(k-1)}{2} = F \quad (\because k > 1 \text{ නිසා})$$

(b) [illegible]

([illegible]) ([illegible])

[illegible]

[illegible] a [illegible]

[illegible]

m [illegible]

$$0 = \frac{1}{2}mu^2 - mga \Rightarrow u^2 = 2ga$$

[illegible]

← $2mu - mu = mv_1 + (2m \times 0)$

$\therefore v_1 = u$

[illegible]

$v_1 + 0 = e(u + u) \Rightarrow e = \frac{1}{2}$ ($\because v_1 = u$ නිසා)

(I) [illegible]

[illegible]

→ $mv_3 + 2mv_2 = mu + 2m \times 0$

$\Rightarrow v_3 + 2v_2 = u$ — ①

[illegible]

$v_2 - v_3 = \frac{1}{2}(u + 0)$ — ②

① + ② න් $3v_2 = \frac{3u}{2} \Rightarrow v_2 = \frac{u}{2}$

එවිට ① න් $v_3 = 0$ [illegible]

[illegible]

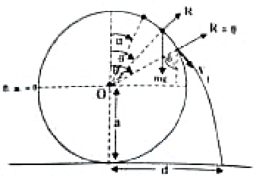
(II) [illegible]

$$= \left(\frac{1}{2} \times m \times 0\right) + \left(\frac{1}{2} \times 2m \times v_2^2\right)$$

([illegible] = 0 [illegible])

$$= mv_2^2 = m\frac{u^2}{4} = \frac{m}{4} \times 2ga = \frac{1}{2}mga$$

03.



[illegible]

(i) $0 + mga\cos\alpha = \frac{1}{2}m(a\dot{\theta})^2 + mga\cos\theta$

$a\dot{\theta}^2 = 2g(\cos\alpha - \cos\theta)$ — ①

(ii) [illegible] $E = mg$ යොදමින්

$mg\cos\theta - R = ma\dot{\theta}^2$

$R = mg\cos\theta - m.2g(\cos\alpha - \cos\theta)$ (① න්)

$R = mg(3\cos\theta - 2\cos\alpha)$ — ②

(iii) [illegible]

$R = 0$ [illegible] $\theta = \theta_1$ [illegible] ② න්

$$3\cos\theta_1 - 2\cos\alpha = 0 \Rightarrow \cos\theta_1 = \frac{2}{3}\cos\alpha$$

[illegible]

[illegible] $= a + a\cos\theta_1$

$= a + a\frac{2}{3}\cos\alpha$

$= a\left[1 + \frac{2}{3}\cos\alpha\right]$ — ③

(iv) $\theta = \theta_1$ [illegible] v_1 [illegible]

$v_1 = a\dot{\theta}_1$ හා $\cos\theta_1 = \frac{2}{3}\cos\alpha$ [illegible]

① න් $a\frac{v_1^2}{a^2} = 2g\left[\cos\alpha - \frac{2}{3}\cos\alpha\right]$

$$v_1^2 = \frac{2}{3}ga\cos\alpha = \frac{2}{3}\cdot ga\cdot\frac{3}{4}$$

$(\because \cos\alpha = \frac{3}{4}$ [illegible])

$$v_1^2 = \frac{1}{2}ga \Rightarrow v_1 = \sqrt{\frac{ga}{2}}$$

[illegible] $\cos\theta_1 = \frac{2}{3} \times \frac{3}{4} = \frac{1}{2} \Rightarrow \theta_1 = \frac{\pi}{3}$ වේ.

[illegible]

→ [illegible] $= v_1\cos\pi/3 = v_1/2$

↓ [illegible] $= v_1\sin\pi/3 = \sqrt{3}v_1/2$

[illegible] $= a\left[1 + \frac{2}{3} \times \frac{3}{4}\right]$

③ ට අනුව

$$= \frac{3a}{2}$$

↓ $s = ut + \frac{1}{2}at^2$ යෙදීමෙන් $\frac{3a}{2} = \frac{\sqrt{3}v_1}{2}t_1 + \frac{1}{2}gt_1^2$

$gt_1^2 + \sqrt{3}v_1t_1 - 3a = 0$ — ④

[illegible]

→ $d - \frac{a\sqrt{3}}{2} = \frac{v_1}{2}t_1 \Rightarrow t_1 = \frac{2d - a\sqrt{3}}{v_1}$ — ⑤

④ හා ⑤ න් t_1 [illegible]

$$g\left(\frac{2d - a\sqrt{3}}{v_1}\right)^2 + \sqrt{3}\left(\frac{2d - a\sqrt{3}}{?}\right) - 3a = 0$$

$$g(4d^2 - 4\sqrt{3}ad + 3a^2) + v_1^2(2\sqrt{3}d - 3a - 3a) = 0$$

$$4gd^2 - 4\sqrt{3}adg + 3ga^2 + \frac{ga}{2}(2\sqrt{3}d - 6a) = 0$$

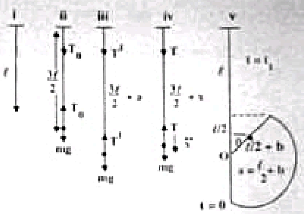
$$\left(\because v_1 = \sqrt{\frac{ga}{2}}\right)$$

$\Rightarrow 4d^2 - 4\sqrt{3}ad + 3a^2 + \sqrt{3}ad - 3a^2 = 0$

$\Rightarrow 4d^2 = 3\sqrt{3}ad$

$\Rightarrow d \neq 0$ නිසා $d = \frac{3\sqrt{3}a}{4}$

04.



λ [illegible]

II න් $T_0 = mg$ — ①

[illegible] $T_0 = \frac{\lambda\,\ell/2}{\ell}$ — ② ([illegible] $\frac{\ell}{2}$ [illegible])

① හා ② න් $mg = \frac{\lambda}{2}$

$\lambda = 2mg$

IV [illegible]

$E = mg$ යොදමින්

$mg - T = m\ddot{x}$

$$T = \frac{\lambda(\ell/2 + x)}{\ell}$$

$$mg - \frac{2mg}{\ell}\left(\frac{\ell}{2} + x\right) = m\ddot{x} \quad (\because \lambda = 2mg \text{ නිසා})$$

$$mg - mg - \frac{2mg}{\ell}x = m\ddot{x}$$

$$\ddot{x} = -\frac{2g}{\ell}x$$

$$\frac{d^2x}{dt^2} + \omega^2x = 0\ ;\ \omega^2 = \frac{2g}{\ell}\ \text{[illegible]}$$

[illegible]

[illegible] O [illegible] $\ddot{x} = 0$ [illegible] $x = 0$ [illegible] II [illegible]

$\therefore -\frac{\ell}{2} \le x \le a$ [illegible]

i. x [illegible]

$x < \ell/2$ [illegible]

[illegible] $2\pi/\omega$

(iv) $a = \frac{\ell}{2} + b \;(b > 0)$

[illegible] O [illegible] $\ell/2$ [illegible] ($\because$ [illegible]) [illegible] $\frac{\ell}{2} + b$ වේ.

(v) [illegible] t_1 නම්

$t_1 = \frac{\pi - \theta}{\omega}$ වේ. $\cos\theta = \frac{\ell/2}{\ell/2 + b} = \frac{\ell}{\ell + 2b}$

$$= \frac{1}{\sqrt{\frac{2g}{\ell}}}\left(\pi - \cos^{-1}\left(\frac{\ell}{\ell + 2b}\right)\right)$$

$$= \sqrt{\frac{\ell}{2g}}\left(\pi - \cos^{-1}\frac{\ell}{\ell + 2b}\right)$$

[illegible] ([illegible])

විසඳුම $x = A\cos\omega t + B\sin\omega t$ ලෙස දී ඇත.

t විෂයයෙන් අවකලනයෙන් $\dot{x} = -A\omega\sin\omega t + B\omega\cos\omega t$

$t = 0$ විට $\dot{x} = 0$ ද $x = a$ ද වේ.

$\therefore a = A\cos(\omega \times 0) + B\sin(\omega \times 0) \Rightarrow A = a$

හා $0 = -a\omega\sin(\omega \times 0) + B\omega\cos(\omega \times 0) \Rightarrow B = 0$

එවිට $x = a\cos\omega t$, [illegible]

$T > 0$ [illegible]

I $a < \frac{\ell}{2}$ නම්,

[illegible] $= \frac{\ell}{2} + a\cos\omega t$ වේ.

[illegible] t සඳහා $-1 \le \cos\omega t \le 1$ බැවින් හා $a < \frac{\ell}{2}$ බැවින්

$\frac{\ell}{2} + a\cos\omega t > 0$ වේ. [illegible] t සඳහා $T > 0$ වේ.

$\therefore$ [illegible] a ද [illegible] $2\pi/\omega$ ද වේ.

II $a = \frac{\ell}{2} + b$ නම් $x = (\frac{\ell}{2} + b)\cos\omega t$

t කාලයේ දී [illegible] $= \frac{\ell}{2} + (\frac{\ell}{2} + b)\cos\omega t$

$t = t_1$ කාලයේ දී [illegible] $T = 0$ වේ.

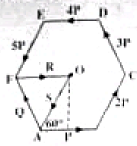
$\frac{\ell}{2} + (\frac{\ell}{2} + b)\cos\omega t_1 = 0$ වේ. ($\because$ [illegible] $= 0$ විට $T = 0$ බැවින්)

$$\Rightarrow \omega t_1 = \cos^{-1}\left(-\frac{\ell}{\ell + 2b}\right) \Rightarrow \omega t_1 = \pi - \cos^{-1}\left(\frac{\ell}{\ell + 2b}\right)$$

$$\Rightarrow t_1 = \frac{1}{\omega}\left[\pi - \cos^{-1}\left(\frac{\ell}{\ell + 2b}\right)\right]$$

05. (i) [illegible]

O වටා [illegible]



$(P + 2P + 3P + 4P + 5P)\, a\sin 60 = Q \times a\sin 60$

$\therefore Q = 15P$

[illegible]

$\rightarrow P + R - 4P + 2P\cos 60 - 3P\cos 60 - 5P\cos 60$
$Q\cos 60 - S\cos 60 = 0$

$\Rightarrow R = 6P + (Q + S)\frac{1}{2}$

$\uparrow 2P\cos 30 + Q\cos 30 - 5P\cos 30 + 3P\cos 30$
$- S\cos 30 = 0$

$\Rightarrow Q = S$

$\therefore Q = S = 15P$, $R = 21P$

II [illegible]

$(P + 2P + 3P + 4P + 5P - Q)\, a\sin 60 = Pa\sqrt{3}$

$\Rightarrow (15P - Q)\, a\frac{\sqrt{3}}{2} = Pa\sqrt{3} \Rightarrow Q = 13P$

[illegible] බැවින්

$Q = S = 13P$ හා

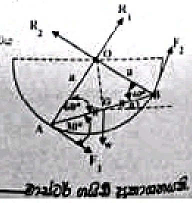
$R = 6P + 13P = 19P$ වේ.

(ii) AB දණ්ඩ [illegible] බැවින්
$F_1 = \mu R_1$ හා $F_2 = \mu R_2$

$OA = OB = AB = a$ බැවින්

$O\hat{A}B = \pi/3$

දණ්ඩේ සමතුලිතතාවය සඳහා B වටා ඝූර්ණ ගැනීමෙන්



අ. පො. ස. (උසස් පෙළ) විභාගය - 2007 අගෝස්තු
සංයුක්ත ගණිතය II
ආදර්ශ පිළිතුරු

$W \times \frac{a}{2}\cos\theta + F_1 \times a\sin 30° = R_1 \times a\sin 60°$

$\frac{W\cos\theta}{2} + \mu R_1 \times \frac{1}{2} = R_1 \times \frac{\sqrt{3}}{2}$

$W\cos\theta = R_1(\sqrt{3} - \mu)$

$R_1 = \frac{W\cos\theta}{(\sqrt{3} - \mu)}$

[illegible] A වටා ඝූර්ණ ගැනීමෙන්

$W \times \frac{a}{2}\cos\theta = R_2 \times a\sin 60 + \mu R_2\, a\sin 30$

$R_2 = \frac{W\cos\theta}{\sqrt{3} + \mu}$ [illegible]

O වටා ඝූර්ණ ගැනීමෙන්

$F_1 \times a + F_2 \times a = W\sin\theta \times a\sin 60 + W\cos\theta \times 0$

[illegible] W [illegible]

$\mu(R_1 + R_2) = W\sin\theta \times \frac{\sqrt{3}}{2}$

$\mu\left(\frac{W\cos\theta}{(\sqrt{3} - \mu)} + \frac{W\cos\theta}{(\sqrt{3} + \mu)}\right) = W\sin\theta\,\frac{\sqrt{3}}{2}$

$\mu W\cos\theta\left(\frac{\sqrt{3} + \mu + \sqrt{3} - \mu}{(\sqrt{3} - \mu)(\sqrt{3} + \mu)}\right) = W\sin\theta\,\frac{\sqrt{3}}{2}$

$\frac{2\mu}{\sqrt{3}}\left(\frac{2\sqrt{3}}{3 - \mu^2}\right) = \frac{\sin\theta}{\cos\theta}$

$\frac{4\mu}{3 - \mu^2} = \tan\theta$

06. (a) [illegible] AC [illegible]

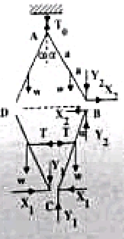
[illegible]

$\therefore Y_1 = 0$

$\therefore$ BC දණ්ඩේ සමතුලිතතාව සඳහා $\downarrow$ [illegible]

$Y_2 = w$

AB දණ්ඩේ සමතුලිතතාව සඳහා A වටා ඝූර්ණ ගැනීමෙන්



$W \times a\sin\alpha + Y_2 \times 2a\sin\alpha = X_2 \times 2a\cos\alpha$

$W\sin\alpha + 2W\sin\alpha = 2X_2\cos\alpha$

$X_2 = \frac{3W}{2}\tan\alpha$

CB දණ්ඩේ සමතුලිතතාවය සඳහා C වටා ඝූර්ණ ගැනීම

$(W \times a\sin\alpha) - (X_2 \times 2a\cos\alpha) -$
$Y_2 \times 2a\sin\alpha + T \times a\cos\alpha = 0$

$T\cos\alpha = 2X_2\cos\alpha + 2Y_2\sin\alpha - W\sin\alpha$

$T = 2X_2 + 2Y_2\tan\alpha - W\tan\alpha$

$= 3W\tan\alpha + 2W\tan\alpha - W\tan\alpha$

$T = 4W\tan\alpha$

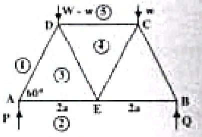
BC දණ්ඩට [illegible]

$\leftarrow X_1 + X_2 = T$

$X_1 + \frac{3W\tan\alpha}{2} = 4W\tan\alpha$

$\Rightarrow X_1 = \frac{5W\tan\alpha}{2}$ හා $Y_1 = 0$

(b)



B වටා ඝූර්ණ ගැනීමෙන්

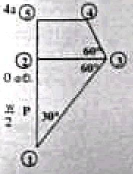
$w \times a + (W - w)\,3a = P \times 4a$

$\therefore P = \frac{3W}{4} - \frac{w}{2}$

$W > 2w$ බැවින් $P = \frac{3W}{4} - \frac{2w}{4} > 0$ වේ.

$P = \frac{3W}{4} - \frac{w}{2} = W - w - \frac{W}{4} + \frac{w}{2}$

$= W - w - \left(\frac{W - 2w}{4}\right)$

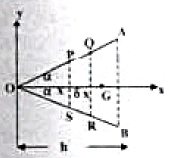


$W > 2w$ බැවින් $\frac{W - 2w}{4} > 0$ වේ.

$\therefore P < W - w$ වේ.

දණ්ඩ	ප්‍රත්‍යාබලය	විශාලත්වය
AE	ආතතිය	$\left(\frac{3W}{4} - \frac{w}{2}\right)\frac{1}{\sqrt{3}}$
DE	තෙරපුම	$\left(\frac{W}{2} - w\right)\frac{1}{\sqrt{3}}$
DC	තෙරපුම	$\frac{W}{2\sqrt{3}}$

07.



සමමිතිකත්වයෙන් කුහර කේතුවේ ගුරුත්ව කේන්ද්‍රය x අක්ෂය මත පිහිටයි. එය G නම් G = ($\bar{X}$, 0) ආකාරය වේ.

y අක්ෂයේ සිට x දුරින් ... PQRS [illegible] සලකමු.

එම [illegible] ස්කන්ධය $= 2\pi x \tan\alpha \times \delta x \sec\alpha \times \sigma$

∴ කේතුවේ ස්කන්ධය $= \int_0^h 2\pi x \tan\alpha \, dx \sec\alpha \, \sigma$

$= 2\pi \tan\alpha \sec\alpha \, \sigma \left(\frac{x^2}{2}\right)_0^h$

$= \pi h^2 \tan\alpha \sec\alpha \, \sigma$

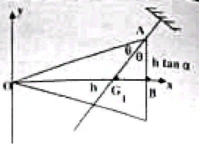
ස්කන්ධ කේන්ද්‍රයේ [illegible]

$\pi h^2 \tan\alpha \sec\alpha \, \sigma \times \bar{x} = \int_0^h 2\pi x \tan\alpha \, dx \, . \sec\alpha \, \sigma . x$

$= 2\pi \tan\alpha \sec\alpha \, \sigma \left(\frac{x^3}{3}\right)_0^h$

$\pi h^2 \tan\alpha \sec\alpha \, \sigma \, \bar{x} = 2\pi \frac{h^3}{3} \tan\alpha \sec\alpha \, \sigma$

$\bar{x} = \frac{2}{3}h$



සමමිතිකත්වයෙන් [illegible] x අක්ෂය මත පිහිටයි. එය G_1 නම් $G_1 = (\bar{\bar{x}}, 0)$ ආකාරය වේ.

[illegible] y අක්ෂය [illegible]

$\bar{\bar{x}}\,[\pi h^2 \tan\alpha \sec\alpha \, \sigma + \pi h^2 \tan^2\alpha \, \sigma] = (\pi h^2 \tan\alpha \sec\alpha \, \sigma)\frac{2h}{3} + (\pi h^2 \tan^2\alpha \, \sigma)h$

$\pi h^2 \tan\alpha \, \sigma$ [illegible]

$\bar{\bar{x}}\,[\sec\alpha + \tan\alpha] = (\frac{2}{3}\sec\alpha + \tan\alpha)h$

$$\bar{\bar{x}} = \frac{(\frac{2}{3}\sec\alpha + \tan\alpha)h}{\sec\alpha + \tan\alpha}$$

AG_1 යනු OAB කෝණයේ සමච්ඡේදකය බැවින්

$OG_1 : G_1B = AO : AB$

$$\frac{OG_1}{G_1B} = \frac{AO}{AB}$$

$$\frac{\bar{\bar{x}}}{h - \bar{\bar{x}}} = \frac{h \sec\alpha}{h \tan\alpha}$$

[illegible]

$$\frac{\bar{\bar{x}}}{h - \bar{\bar{x}} + \bar{\bar{x}}} = \frac{h\sec\alpha}{h\sec\alpha + h\tan\alpha}$$

$$\frac{(\frac{2}{3}\sec\alpha + \tan\alpha)h}{(\sec\alpha + \tan\alpha)h} = \frac{\sec\alpha}{(\sec\alpha + \tan\alpha)}$$

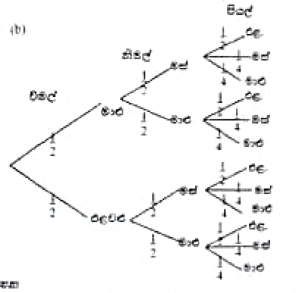
$\tan\alpha = \frac{1}{3}\sec\alpha \Rightarrow \sin\alpha = \frac{1}{3}$

08. (a) $P(A/B) = \frac{P(A \cap B)}{P(B)}$, $P(B) > 0$ විට

i. $P(A/B) = 0 \Rightarrow P(A \cap B) = 0$,
$P(B) > 0$ නිසා
[illegible] $(A \cap B)$ [illegible]
[illegible] A හා B [illegible]

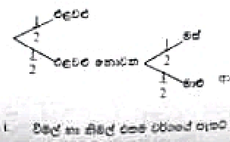
ii. $P(A/B) = P(A) \Rightarrow P(A) = \frac{P(A \cap B)}{P(B)} \Rightarrow$
$P(A \cap B) = P(A). P(B)$
[illegible] A හා B [illegible]

(b)



[illegible]

[illegible] $\frac{1}{2} \times \frac{1}{2} = \frac{1}{4}$ [illegible]



i. [illegible]
$= \frac{1}{2} \times \frac{1}{2} = \frac{1}{4}$

ii. [illegible]
$= \left(\frac{1}{2} \times \frac{1}{4}\right) + \left(\frac{1}{2} \times \frac{1}{4}\right) = \frac{1}{4}$

iii. [illegible]
$= \frac{1}{2} \times \frac{1}{2} \times \frac{1}{4} = \frac{1}{16}$

iv. [illegible]

$= \left(\frac{1}{2} \times \frac{1}{2} \times \frac{1}{2}\right) + \left(\frac{1}{2} \times \frac{1}{2} \times \frac{1}{4}\right) + \left(\frac{1}{2} \times \frac{1}{2} \times \frac{1}{4}\right)$

$= \frac{1}{8} + \frac{1}{16} + \frac{1}{16} = \frac{1}{4}$

(c) A : [illegible]
B : [illegible]

$P(A) = \frac{70}{100}$ හා $P(B/A) = 1$

$P(A') = \frac{30}{100}$ හා $P(B/A') = \frac{1}{5}$ [illegible]

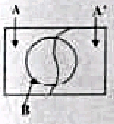
i. [illegible]

[illegible]

$P(B) = P(B/A)P(A) + P(B/A')P(A')$

$= 1 \times \frac{70}{100} + \frac{1}{5} \times \frac{30}{100}$

$= \frac{70}{100} + \frac{6}{100} = \frac{76}{100}$



ii. [illegible]

$$P(A/B) = \frac{P(B/A)P(A)}{P(B/A)P(A) + P(B/A')P(A')}$$

$$= \frac{1 \times \frac{70}{100}}{\frac{76}{100}} = \frac{70}{76} = \frac{35}{38}$$

8. (a)

ප්‍රාන්තරයේ මධ්‍ය අගය	15	30	45	60	75	90
සංඛ්‍යාතය	10	x_1	25	30	x_2	10

මේ අනුව පන්ති ප්‍රාන්තර වන්නේ (7.5 - 22.5)

(22.5 - 37.5) (37.5 - 52.5) ..... ආදී වශයෙන් වේ

සමූහිත දත්ත සඳහා

මාතය $= L_1 + \dfrac{\Delta_1}{\Delta_1 + \Delta_2} C$

මාතය 55 බැවින් එය අයත් වන්නේ 4 වන පන්තියට එනම් 52.5 - 67.5 වේ.

$\therefore L_1 = 52.5,\ \Delta_1 = 30 - 25 = 5\ \Delta_2 = 30 - x_2,\ C = 15$ වේ.

$$55 = 52.5 + \left(\frac{5}{5 + 30 - x_2}\right) 15$$

$$2.5 = \frac{75}{35 - x_2} \Rightarrow 87.5 - 2.5\,x_2 = 75$$

$$\Rightarrow 2.5\,x_2 = 12.5 \Rightarrow \underline{x_2 = 5}$$

මධ්‍යස්ථය 49.5 බැවින්, එය පිහිටන්නේ 3 වන පන්තියේය

$$\text{මධ්‍යස්ථය} = L_1 + \frac{\frac{n}{2} - F}{f_m} C$$

$L_1 = 37.5,\ f_m = 25,\ C = 15,\ F = 10 + x_1$

$$\frac{n}{2} = \frac{10 + x_1 + 25 + 30 + x_1 + 10}{2} = 40 + \frac{x_1}{2}$$

$$\therefore 49.5 = 37.5 + \left(\frac{40 + \frac{x_1}{2} - (10 + x_1)}{25}\right) \times 15$$

$$12 = \frac{3}{5}\left(30 - \frac{x_1}{2}\right) \Rightarrow 20 = 30 - \frac{x_1}{2} \Rightarrow x_1 = 20$$

මධ්‍ය අගය	සංඛ්‍යාතය	අපගමනය d_i	u_i	f_iu_i	$f_iu_i^2$
15	10	-45	-3	-30	90
30	20	-30	-2	-40	80
45	25	-15	-1	-25	25
60	30	0	0	0	0
75	5	15	1	5	5
90	10	20	2	20	40
එකතුව	100			-70	240

මධ්‍යන්‍යය $= A + C\dfrac{\sum f_iu_i}{\sum f_i}$

$= 60 + \dfrac{15 \times (-70)}{100}$

$= \underline{49.5}$

විචලතාව $= C^2\left\{\dfrac{1}{\sum f_i}\sum f_iu_i^2 - \left(\dfrac{\sum f_iu_i}{\sum f_i}\right)^2\right\}$

$= 225\left\{\dfrac{240}{100} - \left(\dfrac{-70}{100}\right)^2\right\}$

$= 225\,[\,2.4 - 0.49\,]$

$= \underline{429.75}$

(b)

	I කුලකය	II කුලකය
සංඛ්‍යාතය	$n = 12$	$m = 20$
මධ්‍යන්‍යය	$\bar{x} = 4$	$\bar{y} = 5$
විචලතාව	$\sigma_x = 2$	$\sigma_y = 3$

සංයුක්ත කුලකයේ මධ්‍යන්‍යය $\bar{Z}$ නම්

$$\bar{z} = \frac{n\bar{x} + m\bar{y}}{n + m} \text{ නිසා } \bar{z} = \frac{(12 \times 4) + (20 \times 5)}{12 + 20}$$

$$= \frac{148}{32} = \underline{4.625}$$

සංයුක්ත කුලකයේ විචලතාව σ^2 නම්

$$\sigma^2 = \frac{\sum_{i=1}^{12} x_i^2 + \sum_{i=1}^{20} y_i^2}{m + n} - (\bar{z})^2$$

$\sum_{i=1}^{12} x_i^2 = n(\sigma_x^2 + \bar{x}^2)$ $\qquad$ $\sum_{i=1}^{20} y_i^2 = m(\sigma_y^2 + \bar{y}^2)$

$= 12\,(4 + 16)$ $\qquad$ $= 20\,(25 + 9)$

$= 240$ $\qquad$ $= 680$

$$\therefore \sigma_z^2 = \frac{240 + 980}{12 + 20} - (4.625)^2$$

$= 7.359$

$\therefore$ සම්මත අපගමනය (සංයුක්ත කුලකයේ)

$= \sqrt{7.359} \simeq \underline{2.712}$

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