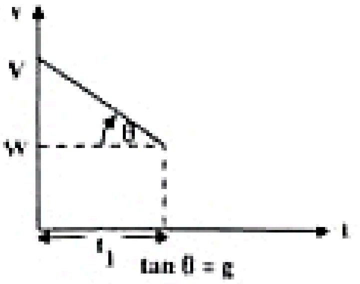
8. (a) (i) $0 \le t \le t_1$ [illegible]

(ඔ ප E) = ↑U ; (ඔ ත E) = 0

(P ප ඔ) = ↑V ; (P ත E) = ↓g

$\tan\theta = \dfrac{V - W}{t_1} \rightarrow t_1 = \dfrac{V - W}{g} \therefore W = V - gt_1$




(ii) $t_1 \le t \le t_2$ [illegible]

(P ත E) = ↓g  (Q ත E) = ↓g

(Q ත P) = (Q ත E) + (E ත P)

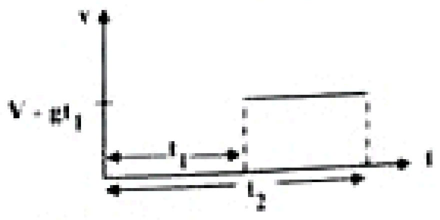
= ↓g + ↑g = 0

∴ (Q ප P) [illegible] (Q ප P) = ↑ $(V - gt_1)$

$t = t_1$ [illegible]

(P ප Q) = (P ප E) = ½ (V + W) $t_1$

= ½ $(V + V - gt_1)\, t_1$




$t = t_1$ [illegible]

(Q ප P) = (Q ප ඔ) + (ඔ ප P)

= ↑V + ↓W

= ↑V - W = ↑V - $(V - gt_1)$

= ↑ $gt_1$

$t_2 - t_1$ [illegible] = $gt_1(t_2 - t_1)$

$t = t_2$ [illegible] P [illegible] Q [illegible]

[illegible] (Q ප P) = 0 [illegible]

$gt_1(t_2 - t_1) - \dfrac{1}{2}(2V - 2gt_1)t_1 = 0$

$gt_1(t_2 - t_1) = \dfrac{t_1}{2}(2V - gt_1)$

$t_2 - t_1 = \dfrac{1}{2g}(2V - gt_1)$ ($t_2 > t_1$ [illegible])

$t_2 = \dfrac{V}{g} - \dfrac{t_1}{2} + t_1$

$= \dfrac{V}{g} + \dfrac{t_1}{2}$

$t = t_2$ [illegible]

(Q ප E) = $U + V - g(t_2 - t_1)$

$= U + V - g\left(\dfrac{V}{g} + \dfrac{t_1}{2} - t_1\right)$

$= U + \dfrac{gt_1}{2}$

(P ප E) = (P ප Q) + (Q ප E)

$= -gt_1 + U + \dfrac{gt_1}{2}$

$= U - \dfrac{gt_1}{2}$

(b) (ක E) = U  (ක E) = ↑V

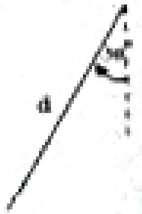
(ක [illegible]) = (ක E) + (E [illegible])

= u + ↓v

= ↓v + u

= $\overrightarrow{PQ} + \overrightarrow{QR}$

= $\overrightarrow{PR}$



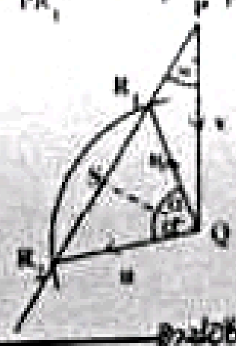

Q [illegible] u [illegible] (u < v < 2u) [illegible] P [illegible] $R_1$ [illegible] $R_2$ [illegible] $\overrightarrow{QR_1}$ [illegible] $\overrightarrow{QR_2}$ [illegible]

[illegible] $R_1\hat{Q}R_2 = 2\alpha$ [illegible]

[illegible] $\cos\alpha = \dfrac{v\sin 30}{u} = \dfrac{v}{2u}$ [illegible]

[illegible] $t_1$ [illegible] $t_2$ [illegible]

$t_1 = \dfrac{d}{PR_1}$ [illegible] $t_2 = \dfrac{d}{PR_2}$ [illegible]



කාල ප්‍රාන්තරය $t_1 - t_2 = \frac{d}{PR_1} - \frac{d}{PR_2}$

$= d\left[\frac{PR_2 - PR_1}{PR_1 \cdot PR_2}\right]$

$= d\frac{[PS + SR_2 - (PS - SR_1)]}{(PS - SR_1)\ (PS + SR_2)}$

$= \frac{d \cdot 2SR_1}{PS^2 - SR_1^2}$

($\because SR_1 = SR_2$ බැවින්)

$t_1 - t_2 = \frac{2d\sqrt{u^2 - (v/2)^2}}{(v\sqrt{3}/2)^2 - [u^2 - (v/2)^2]}$

$= \frac{2d \frac{1}{2}\sqrt{4u^2 - v^2}}{\frac{1}{4}(3v^2 - 4u^2 + v^2)}$

$= \frac{d\sqrt{4u^2 - v^2}}{v^2 - u^2}$

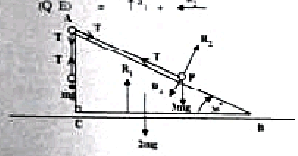
02. (a) (P, ඍ) = ($a_1$, 30° ආනතව)

(Q, ඍ) = $\uparrow a_1$

(ඍ, E) = $\leftarrow a_2$

(P, E) = ($a_1$, 30° ආනතව) + $\leftarrow a_2$

(Q, E) = $\uparrow a_1 + \leftarrow a_2$




E = mg යොදමු.

$\uparrow$ Q : $T - mg = ma_1$ ⟶ ①

P ; $3\,mg \sin 30^0 - T = 3m(a_1 - a_2 \cos 30^0)$ ——— ②

$\leftarrow$ පද්ධතිය ; $0 = 2ma_2 + m.a_2 + 3m(a_2 - a_1 \cos 30)$ ——— ③

① + ② $\frac{mg}{2} = 4ma_1 - 3ma_2\frac{\sqrt{3}}{2}$ ——— ④

③ න්, $0 = 6ma_2 - 3ma_1\frac{\sqrt{3}}{2}$ ——— ⑤

⑤ න් $a_1 = \frac{4a_2}{\sqrt{3}}$

④ න් $\frac{g}{2} = 4 \times \frac{4a_2}{\sqrt{3}} - \frac{3\sqrt{3}a_2}{2}$

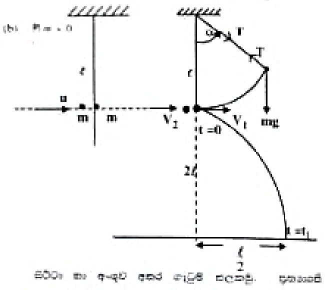
$\sqrt{3}g = 32\,a_2 - 9\,a_2$

$a_2 = \frac{\sqrt{3}g}{23}$

$\therefore a_1 = \frac{4}{\sqrt{3}} \times \frac{\sqrt{3}g}{23} = \frac{4g}{23}$

$T = mg + \frac{4mg}{23} = \frac{27mg}{23}$

(b)




ගැටුම සහ ආඝාතය අතර ගැටුම සලකමු. ගම්‍යතා සංස්ථිති නියමයෙන්,

$mu = mv_1 + mv_2$ ⟶ ①

නිව්ටන්ගේ ප්‍රත්‍යාගති නියමයෙන්,

$eu = v_1 - v_2$ ⟶ ②

① හා ② විසඳීමෙන්

$v_1 = \frac{u}{2}(1 + e)$ හා $v_2 = \frac{u}{2}(1 - e)$

අංශුවේ චලිතය සලකා $S = ut + \frac{1}{2}at^2$ යෙදීමෙන්

$\rightarrow \frac{\ell}{2} = v_2 t_1$ ——— ③

$\downarrow 2\ell = \frac{1}{2}g t_1^2$ ——— ④

$t_1$ ඉවත් කිරීමෙන් $16\,v_2^2 = g\,\ell$ ——— ⑤

පද්ධතිය සඳහා ශක්ති සංස්ථිති නියමය යෙදීමෙන්

$\frac{1}{2}m v_1^2 - mg\ell = 0 - mg\ell\cos\alpha$

$v_1^2 = 2g\ell(1 - \cos\alpha)$ ——— ⑥

$\frac{⑥}{⑤}$ න් $\frac{v_1^2}{16v_2^2} = \frac{2g\ell(1 - \cos\alpha)}{g\ell}$





$\Rightarrow \frac{v_1^2}{v_2^2} = 32 \times 2\sin^2\frac{\alpha}{2}$

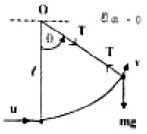
$\frac{v_1}{v_2} = \frac{1 + e}{1 - e}$ බැවින් $\frac{(1 + e)^2}{(1 - e)^2}$

$= 64\sin^2\frac{\alpha}{2}$

$\Rightarrow \frac{1 + e}{1 - e} = 8\sin\frac{\alpha}{2}$ ($\because 0 < e < 1$ නිසා)

$\therefore e = \frac{8\sin\frac{\alpha}{2} - 1}{8\sin\frac{\alpha}{2} + 1}$

03. අංශුවේ චලිතය සලකා ශක්ති සංස්ථිති නියමය යෙදීමෙන්,




$\frac{1}{2}mu^2 - mg\ell = \frac{1}{2}mv^2 - mg\ell\cos\theta$

$v^2 = u^2 - 2g\ell(1 - \cos\theta)$ ——— ①

E = mg යෙදීමෙන්,

$T - mg\cos\theta = m\frac{v^2}{\ell}$ ——— ②

ආදේශයෙන්,

(i) $T = mg\cos\theta + \frac{m}{\ell}[u^2 - 2g\ell + 2g\ell\cos\theta]$

$= \frac{m}{\ell}[u^2 - 2g\ell + 3g\ell\cos\theta]$

$T = m[3g\cos\theta - 2g + \frac{u^2}{\ell}]$

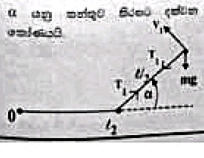
(ii) අංශුව යන්තමින් O හි මට්ටමට ළඟාවේ නම්,

$\theta = \frac{\pi}{2}$ විට $v \geq 0$ විය යුතු ය

① න් $u^2 - 2g\ell \geq 0 \Rightarrow u^2 \geq 2g\ell$

$\therefore$ අවම $u = \sqrt{2g\ell}$

(iii) α යනු තන්තුව තිරසට දක්වන ආනතිය කෝණයයි.




අංශුවේ චලිතය සලකා, ශ. සං. නි. යෙදීමෙන්,

$\frac{1}{2}m(u^2 - 2g\ell) + 0 = \frac{1}{2}mv_1^2 + mg\frac{\ell}{2}\sin\alpha$

$u^2 - 2g\ell = v_1^2 + g\ell\sin\alpha$

$v_1^2 = u^2 - 2g\ell - g\ell\sin\alpha$

E = mg යෙදීමෙන්,

$T_1 + mg\sin\alpha = \frac{m v_1^2}{\ell/2}$

$T_1 = \frac{2m}{\ell}(u^2 - 2g\ell - g\ell\sin\alpha) - mg\sin\alpha$

තන්තුව බුරුල් වන විට $T_1 = 0$

$\Rightarrow \frac{2m}{\ell}(u^2 - 2g\ell - g\ell\sin\alpha) - mg\ell\sin\alpha = 0$

$2u^2 - 4g\ell - 2g\ell\sin\alpha - g\ell\sin\alpha = 0$

$\sin\alpha = \frac{2u^2 - 4g\ell}{3g\ell}$ වේ.

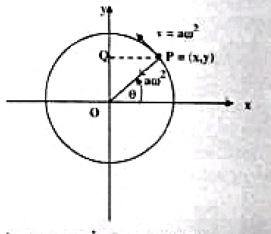
$0 < \sin\alpha < 1$ නිසා

$0 < \frac{2u^2 - 4g\ell}{3g\ell} < 1 \Rightarrow 2g\ell < u^2 < \frac{7g\ell}{2}$

04. $OQ = a\sin\theta$

$\therefore y = a\sin\theta$ ——— ①

t විෂයයෙන් අවකලනයෙන්,




$\dot{y} = a\cos\theta\,\dot{\theta} = a\omega\cos\theta$ ($\because \dot{\theta} = \frac{d\theta}{dt} = \omega$)

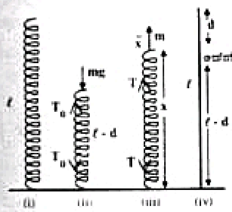
$\therefore \ddot{y} = -a\omega^2\sin\theta$

$\ddot{y} = -\omega^2 y$ (① න්)

තවද $-a \leq y \leq a$ වේ.

∴ P හි y අක්ෂය මත ප්‍රක්ෂේපණය වන Q , [illegible]

∴ [illegible] $= \frac{2\pi}{\omega}$ වේ.




(ii) [illegible]

$T_0 = mg$ ——— ①

$T_0 = \frac{\lambda d}{\ell}$ ——— ②

([illegible] λ [illegible])

$\therefore mg = \frac{\lambda d}{\ell} \Rightarrow \frac{\lambda}{\ell} = \frac{mg}{d}$

(iii) [illegible]

$T = \frac{\lambda(\ell - x)}{\ell} = \frac{mg}{d}(\ell - x)$

F = ma යෙදීමෙන්, $\uparrow T - mg = m\ddot{x}$

$\frac{mg}{d}(\ell - x) - mg = m\ddot{x}$

$-\frac{g}{d}(x - \ell + d) = \ddot{x}$

[illegible] x = 0 [illegible] x = ℓ - d [illegible]

**[illegible]**

[illegible] a [illegible] t [illegible] y [illegible]

$v^2 = \omega^2(a^2 - y^2)$

[illegible] h [illegible]

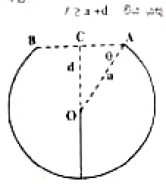
[illegible] $v_1$ [illegible] $v_1^2 = 2gh$ [illegible]

y = d විට $v = v_1$ [illegible]

$2gh = \frac{g}{d}(a^2 - d^2) \Rightarrow a^2 = 2dh + d^2$

∴ [illegible] $a = \sqrt{2dh + d^2}$ [illegible]

[illegible] d < a [illegible]
(iv [illegible])
ℓ ≥ a + d [illegible]




[illegible]

$\hat{OAC} = \theta$ නම්

$\tan\theta = \frac{d}{\sqrt{a^2 - d^2}}$

[illegible] T නම්

$T = 2\left(\frac{\pi/2 + \theta}{\omega}\right)$

$= 2\sqrt{\frac{d}{g}}\left(\frac{\pi}{2} + \tan^{-1}\frac{d}{\sqrt{a^2 - d^2}}\right)$

$= \sqrt{\frac{d}{g}}\left(\pi + 2\tan^{-1}\frac{d}{\sqrt{2dh + d^2 - d^2}}\right)$

$= \sqrt{\frac{d}{g}}\left(\pi + 2\tan^{-1}\sqrt{\frac{d}{2h}}\right)$

[illegible] $\frac{3\pi}{2}\sqrt{\frac{d}{g}}$ [illegible]

$\sqrt{\frac{d}{g}}\left(\pi + 2\tan^{-1}\sqrt{\frac{d}{2h}}\right) \geq \frac{3\pi}{2}\sqrt{\frac{d}{g}} \Rightarrow \tan^{-1}\sqrt{\frac{d}{2h}} \geq \frac{\pi}{4}$

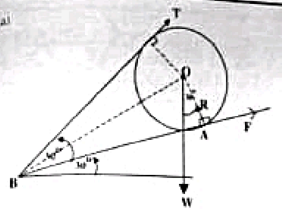
$\Rightarrow \sqrt{\frac{d}{2h}} \geq 1 \Rightarrow \frac{h}{d} \leq \frac{1}{2}$

∴ [illegible] $\frac{h}{d} = \frac{1}{2}$





05. (a)




[illegible] a නම්. [illegible] O [illegible]

$\circlearrowleft$ F × a = T × a ——— ①

A [illegible]

$\circlearrowleft$ W × a sin 30 = T × a tan 75 × sin 30

$W = T\left(\frac{\sqrt{3}+1}{\sqrt{3}-1}\right)\left(\frac{\sqrt{3}+1}{\sqrt{3}+1}\right)$

$W = T\frac{(4 + 2\sqrt{3})}{2}$

$T = \frac{W}{2 + \sqrt{3}}$

$T = W(2 - \sqrt{3}) = F$ (① ට අනුව)

$\circlearrowleft$ B R × a tan 75 = W × a sec 75° × cos 45°

$R\frac{\sin 75}{\cos 75} = W\frac{1}{\cos 75} \times \frac{1}{\sqrt{2}}$

$R = \frac{W}{\sqrt{2}\sin 75} = \frac{W}{\sqrt{2}\cos 15}$

[illegible] $\frac{F}{R} \leq \mu$ [illegible]

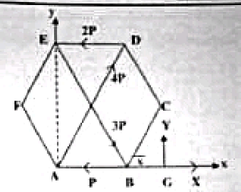
$\frac{W(2 - \sqrt{3})}{\frac{W}{\sqrt{2}\cos 15}} = \sqrt{2}(2 - \sqrt{3})\cos 15 \leq \mu$ [illegible]

(μ [illegible])

(b) [illegible] x හා y [illegible]

$\rightarrow X = -P - 2P + 3P\cos 60 + 4P\cos 60$

$= \frac{P}{2}$




$\uparrow Y = 4P\cos 30 - 3P\cos 30$

$= \frac{P\sqrt{3}}{2}$

$R = \sqrt{X^2 + Y^2} = P$

R [illegible] x [illegible] θ [illegible]

$\tan\theta = \frac{Y}{X} = \sqrt{3} \Rightarrow \theta = 60°$

[illegible] AB [illegible] G [illegible] A [illegible] (BG = $\bar{x}$ [illegible])

$\circlearrowleft$ $2P \times 2a\cos 30 - 3P \times a\sin 60 = \frac{P\sqrt{3}}{2}(a + \bar{x})$

$\bar{x} = 0$ [illegible]

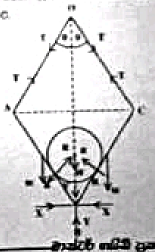
∴ [illegible] P, BC [illegible]

CB [illegible] P [illegible] FE [illegible] P [illegible] FE [illegible] BC [illegible] P [illegible] FE [illegible] P [illegible]

[illegible] AFEDCB [illegible] $P \times 2a\cos 30 = Pa\sqrt{3}$ වේ.

06. (a) [illegible] O [illegible]

∴ Y = 0



බෝලයේ සමතුලිතතාවය සඳහා ↑ විභේදනයෙන්,

$2R\sin\theta = W \Rightarrow R = \frac{W}{2\sin\theta}$

පද්ධතියේ සමතුලිතතාව සඳහා ↑ විභේදනයෙන්,

$2T\cos\theta = 3W \Rightarrow T = \frac{3W}{2\cos\theta}$

AB දණ්ඩේ සමතුලිතතාවය සඳහා, B වටා ඝූර්ණ ගැනීමෙන්,

$R \times \frac{a}{3}\cot\theta + W \times a\sin\theta = (T\cos\theta \times 2a\sin\theta) + (T\sin\theta \times 2a\cos\theta)$

$\frac{W}{2\sin\theta}\cdot\frac{a}{3}\frac{\cos\theta}{\sin\theta} + W \times a\sin\theta = 2 \times \frac{3W}{2\cos\theta} \times 2a\sin\theta\cos\theta$

$\cos\theta + 6\sin^3\theta = 36\sin^3\theta$

$\frac{\cos\theta}{\sin^3\theta} = 30\frac{\sin^3\theta}{\sin^3\theta}$

$\cot\theta\,\mathrm{cosec}^2\theta = 30$

$\cot\theta(1+\cot^2\theta) = 30$

$\cot^3\theta + \cot\theta - 30 = 0$

$(\cot\theta - 3)(\cot^2\theta + 3\cot\theta + 10) = 0$

$\cot\theta = 3$ හෝ $\cot^2\theta + 3\cot\theta + 10 = 0$

මෙහි විවේචකය $\Delta = 9 - 40 < 0$

∴ θ සඳහා ඇත්තේ $\cot^{-1}3$ අගය පමණි.

AB සඳහා තිරස් විභේදනයෙන්,

$T\sin\theta + X = R\cos\theta$

$\frac{3W}{2\cos\theta}\sin\theta + X = \frac{W}{2\sin\theta} \times \cos\theta$

$X = \frac{W}{2}\cot\theta - \frac{3W}{2}\tan\theta$

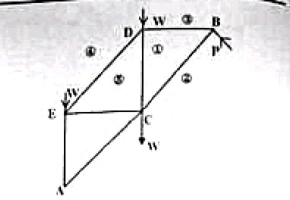
$= \frac{W}{2} \times 3 - \frac{3W}{2} \times \frac{1}{3} = W$

(b) සමාන දණ්ඩක දිග a යැයි සිතමු.
රාමු සැකිල්ලේ සමතුලිතතාව සඳහා A වටා ඝූර්ණ ගැනීමෙන්,

$W \times a + W \times a = P \times 2a\sec 45^\circ$

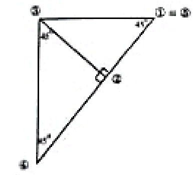
$2W = 2P\sqrt{2}$

$P = \frac{W}{\sqrt{2}}$

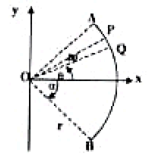



ප්‍රත්‍යාබල සටහන.

| දණ්ඩ | ප්‍රත්‍යාබල වර්ගය | විශාලත්වය |
|---|---|---|
| BD | තෙරපුම | W |
| BC | ආතතිය | W / √2 |
| DE | තෙරපුම | √2 W |




07. සමමිතිකත්වයෙන් චාපයේ ගුරුත්ව කේන්ද්‍රය G, x- අක්ෂය මත පිහිටයි.




$P\hat{O}Q = \delta\theta$ වන පරිදි ඇති කුඩා කොටසක් සලකා බලමු

PQ චාප දිග = $r\delta\theta$

$\rho_1$ යනු ඒකක දිගක ස්කන්ධය නම්,

PQ චාපයේ ස්කන්ධය = $r\delta\theta\rho_1$

PQ චාප කොටසේ ස්කන්ධ කේන්ද්‍රයට y අක්ෂයේ සිට ඇති දුර = $r\cos\theta$

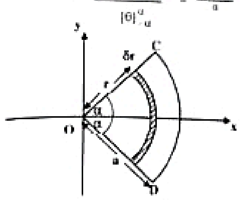




ස්කන්ධ කේන්ද්‍රයේ අර්ථ දැක්වීමට අනුව,

$$\bar{x} = \frac{\int_{-\alpha}^{\alpha} r^2\cos\theta\, d\theta\, \rho_1}{\int_{-\alpha}^{\alpha} r\,d\theta\,\rho_1}$$

$$= \frac{r[\sin\theta]_{-\alpha}^{\alpha}}{[\theta]_{-\alpha}^{\alpha}} = \frac{r\sin\alpha}{\alpha}$$




අරය r හා ඝනකම δr වන කුඩා චාපයක් සලකමු

එහි ස්කන්ධය = $r\,2\alpha\,\delta r\,\rho_2$

මෙහි $\rho_2$ යනු ආස්තරයේ ඒකක වර්ගඵලයක ස්කන්ධය වේ

මෙම කුඩා චාපයේ ස්කන්ධ කේන්ද්‍රයට y අක්ෂයේ සිට ඇති දුර $= \frac{r\sin\alpha}{\alpha}$

∴ ආස්තරයේ ස්කන්ධ කේන්ද්‍රයට y අක්ෂයේ සිට ඇති දුර

$$\bar{x} = \frac{\int_0^a r\,2\alpha\,dr\,\rho_2 \times \frac{r\sin\alpha}{\alpha}}{\int_0^a r\,2\alpha\,dr\,\rho_2}$$

$$= \frac{\sin\alpha}{\alpha}\left\{\frac{\left(\frac{r^3}{3}\right)_0^a}{\left(\frac{r^2}{2}\right)_0^a}\right\} = \frac{2}{3}\frac{a\sin\alpha}{\alpha}$$

OA = a

OC $= \frac{a}{\sqrt{3}}$

AC $= \frac{2a}{\sqrt{3}}$

ABE D $= \frac{\pi a^2}{2}$

ABC Δ $= \frac{1}{2} \times 2a \times \frac{a}{\sqrt{3}}$

ACBD $= \frac{1}{2} \times \left(\frac{2a}{\sqrt{3}}\right)^2 \times \frac{2\pi}{3}$

| වස්තුව | ස්කන්ධය | ස්කන්ධ කේන්ද්‍රයට C සිට දුර |
|---|---|---|
| ABE අර්ධ වෘත්තාකාර ආස්තරය | $\frac{\pi a^2\rho}{2}$ | $\frac{a}{\sqrt{3}} + \frac{4a}{3\pi}$ |
| ABC ත්‍රිකෝණාකාර ආස්තරය | $\frac{a^2\rho}{\sqrt{3}}$ | $\frac{2}{3} \cdot \frac{a}{\sqrt{3}}$ |
| OADB කේන්ද්‍රික ඛණ්ඩය | $\frac{4\pi a^2}{9}\rho$ | $\frac{2a}{\pi}$ |
| ADBE ලෙන්ස | $a^2\rho\left(\frac{\pi}{18} + \frac{1}{\sqrt{3}}\right)$ | $\bar{x}$ |

C වටා ඝූර්ණ ගැනීමෙන්

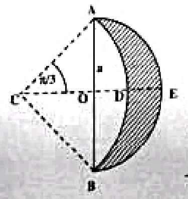
$$\frac{\pi a^2\rho}{2}\left(\frac{a}{\sqrt{3}} + \frac{4a}{3\pi}\right) + \frac{a^2\rho}{\sqrt{3}} \times \frac{2a}{3\sqrt{3}} - \frac{4\pi a^2\rho}{9} \times \frac{2a}{\pi} = a^2\rho\left(\frac{\pi}{18} + \frac{1}{\sqrt{3}}\right)\bar{x}$$

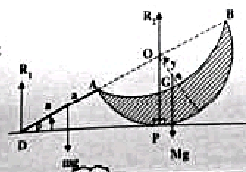
$$\frac{\pi a}{2\sqrt{3}} + \frac{2a}{3} + \frac{2a}{9} - \frac{8a}{9} = \left(\frac{\pi + 6\sqrt{3}}{18}\right)\bar{x}$$

$$\frac{\pi a}{2\sqrt{3}} \times \frac{18}{(\pi + 6\sqrt{3})} = \bar{x}$$

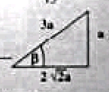
$$\therefore \bar{x} = \frac{3\sqrt{3}\pi a}{(\pi + 6\sqrt{3})} = ka$$

07.




$\bar{y} = ka - \frac{a}{\sqrt{3}}$



[illegible] P [illegible]

$R_1 \times 3a\cos\beta + Mg\,(ka - \frac{a}{\sqrt{3}})\sin\beta - mg\;2a\cos\beta = 0$

[illegible] $R_1$ [illegible] D [illegible] D [illegible] $R_1 > 0$ [illegible]

[illegible] $mg\;2a\cos\beta > Mg\,(ka - \frac{a}{\sqrt{3}})\sin\beta$

$2m > M\frac{(\sqrt{3}k - 1)}{\sqrt{3}}\tan\beta$

$2m > M\frac{(\sqrt{3}k - 1)}{\sqrt{3}} \times \frac{1}{2\sqrt{2}}$

$4\sqrt{6}m > M(\sqrt{3}k - 1)$

08. B [illegible] A [illegible] සම්භාවිතාව

$P(B|A) = P\frac{(A \cap B)}{P(A)}$ [illegible] $P(A) > 0$ [illegible]

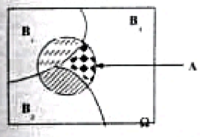
[illegible]

$P(C|A \cap B) = \frac{P(C \cap A \cap B)}{P(A \cap B)}$ [illegible]

$\therefore P(A)\;P(B|A)\;P(C|A \cap B) = P(A).\frac{P(B \cap A)}{P(A)}\;\frac{P(C \cap A \cap B)}{P(A \cap B)}$

$= P(A \cap B \cap C)$

[illegible]




$A = (B_1 \cap A) \cup (B_2 \cap A) \cup (B_3 \cap A)$

$P(A) = P[(B_1 \cap A) \cup (B_2 \cap A) \cup (B_3 \cap A)]$

$= P(B_1 \cap A) + P(B_2 \cap A) + P(B_3 \cap A)$

($\because$ [illegible] $B_1 \cap A$, $B_2 \cap A$, $B_3 \cap A$ [illegible])

$P(A) = P(B_1)P(A|B_1) + P(B_2)P(A|B_2) + P(B_3)P(A|B_3)$ ——— ①

([illegible])

$P(B_i|A) = \frac{P(A \cap B_i)}{P(A)}$ $i = 1, 2, 3$ [illegible]

$= \frac{P(B_i)P(A|B_i)}{P(B_1)P(A|B_1) + P(B_2)P(A|B_2) + P(B_3)P(A|B_3)}$

$X_i$ : [illegible] $(i = 1, 2, 3)$ [illegible]

$Y_i$ : [illegible] $(i = 1, 2, 3)$ [illegible]

$Z_i$ : [illegible] $(i = 1, 2, 3)$ [illegible]

$P(X_i) = \frac{50}{100} = 0.5$, $P(Y_i) = \frac{20}{100} = 0.2$,

$P(Z_i) = \frac{30}{100} = 0.3$

(i) [illegible]
$= Z_1 \cap Z_2 \cap Z_3$

[illegible] $= P(Z_1 \cap Z_2 \cap Z_3)$

$= P(Z_1)P(Z_2)P(Z_3)$

($\because Z_1, Z_2, Z_3$ [illegible])

$= 0.3 \times 0.3 \times 0.3 = 0.027$

(ii) [illegible]
$= (X_1 \cap X_2 \cap X_3) \cup (Y_1 \cap Y_2 \cap Y_3) \cup (Z_1 \cap Z_2 \cap Z_3)$

[illegible]
$= P[(X_1 \cap X_2 \cap X_3) \cup (Y_1 \cap Y_2 \cap Y_3) \cup (Z_1 \cap Z_2 \cap Z_3)]$

$= P(X_1 \cap X_2 \cap X_3) + P(Y_1 \cap Y_2 \cap Y_3) + P(Z_1 \cap Z_2 \cap Z_3)$

($\because$ [illegible])

$= P(X_1)\;P(X_2)\;P(X_3) + P(Y_1)\;P(Y_2)\;P(Y_3) + P(Z_1)\;P(Z_2)\;P(Z_3)$

($\because$ [illegible])

$(0.5)^3 + (0.2)^3 + (0.3)^3 = 0.125 + 0.008 + 0.027$

$0.160 = \underline{\underline{0.16}}$

(iii) [illegible]
$= (X_1 \cap Y_2 \cap Y_3) \cup (Y_1 \cap X_2 \cap Y_3) \cup (Y_1 \cap Y_2 \cap X_3)$

[illegible]
$= P(X_1 \cap Y_2 \cap Y_3) + P(Y_1 \cap X_2 \cap Y_3) + P(Y_1 \cap Y_2 \cap X_3)$





([illegible])

$= P(X_1)P(Y_2)P(Y_3) + P(Y_1)P(X_2)P(Y_3) + P(Y_1)P(Y_2)P(X_3)$

$= 0.5 \times 0.2 \times 0.2 + 0.2 \times 0.5 \times 0.2 + 0.2 \times 0.2 \times 0.5$

$= 3 \times 0.5 \times 0.2 \times 0.2$

$= 0.060$

(iv) [illegible]

$(X_1 \cap Y_2 \cap Z_3) \cup (Y_1 \cap Z_2 \cap X_3) \cup (Z_1 \cap X_2 \cap Y_3)$

[illegible] $= (0.5 \times 0.2 \times 0.3) + (0.2 \times 0.3 \times (0.5) + (0.3 \times 0.5 \times 0.2)$

$= 3 \times 0.5 \times 0.2 \times 0.3$

$= \underline{\underline{0.090}}$

A ; [illegible]

(a) [illegible]

[illegible] $= P[(X_1 \cap X_2 \cap X_3)|A]$

$= \frac{P(X_1 \cap X_2 \cap X_3 \cap A)}{P(A)}$

$= \frac{P(X_1 \cap X_2 \cap X_3)}{P(A)}$

[$\because X_1 \cap X_2 \cap X_3 \cap A = X_1 \cap X_2 \cap X_3$ [illegible]]

$= \frac{0.125}{0.16} = \frac{125}{160}$

$= \frac{25}{32} = 0.78125$

(b) [illegible]

$= \frac{0.008}{0.16} = \frac{1}{20}$

$= 0.05$

(c) [illegible]

$= \frac{0.027}{0.16} = \frac{27}{160}$

$= 0.16875$

[illegible]

09. (a) $\sum_{i=1}^{n}(x_i - \bar{x})^2 = \sum_{i=1}^{n}(x_i^2 - 2\bar{x}\,x_i + \bar{x}^2)$

$= \sum_{i=1}^{n} x_i^2 - 2\bar{x}\sum_{i=1}^{n} x_i + \sum_{i=1}^{n} \bar{x}^2$

$= \sum_{i=1}^{n} x_i^2 - 2\bar{x}\;n\bar{x} + n\bar{x}^2$

$(\because \bar{x} = \frac{1}{n}\sum_{i=1}^{n} x_i)$

$= \sum_{i=1}^{n} x_i^2 - n\bar{x}^2$

පිරිමි ළමුන් 200

[illegible] $x_i$ [illegible] $\sum x_i = 920$

$\sum x_i^2 = 5032$

$n = 200$ [illegible]

මධ්‍යන්‍යය $\bar{x} = \frac{1}{n}\sum_{i=1}^{n} x_i$

$= \frac{1}{200} \times 920$

$= \underline{\underline{4.6}}$

විචලතාව $= \frac{1}{n}\sum_{i=1}^{200} x_i^2 - \bar{x}^2$

$= \frac{1}{200} \times 5032 - (4.6)^2$

$= 25.16 - 21.16 = 4$

$\therefore$ සම්මත අපගමනය $= \underline{\underline{2}}$

<u>ගැහැණු ළමුන් 50</u>

[illegible] $y_i$ [illegible] $\bar{y} = 4.4$

[illegible] $= 2.2$

$\therefore$ විචලතාව $= 4.84$

$\sum_{i=1}^{n} y_i = n\bar{y}$

$= 50 \times 4.4$

$= 220$

[illegible] $4.84 = \frac{1}{50}\sum_{i=1}^{50} y_i^2 - \bar{y}^2$

$= \frac{1}{50}\sum_{i=1}^{50} y_i^2 - (4.4)^2$

$4.84 \times 50 = \sum_{i=1}^{50} y_i^2 - 19.36 \times 50$

$$\sum_{i=1}^{50} y_i^2 = 24.2 \times 50$$

$$= 1210$$

මුළු 250 සැලකූ විට, ලකුණු ගණන $z_i$ නම්,

$$\sum_{i=1}^{250} z_i = 920 + 220$$

$$= 1140$$

$$\therefore \bar{z} = \frac{1140}{250}$$

$$= \underline{4.56}$$

$$\text{විචලතාව} = \frac{1}{250}\sum_{i=1}^{n} z_i^2 - \bar{z}^2$$

$$= \frac{1}{250}[5032 + 1210] - (4.56)^2$$

$$= 24.968 - 20.7936 = 4.1744$$

$\therefore$ සම්මත අපගමනය $= \sqrt{4.1744} \approx \underline{2.04}$

(b) මුල් ලකුණ $x$ ද, පරිවර්තිත ලකුණ $y$ ද යැයි ගනිමු.
$a$ හා $b$ නියත විට, රේඛීය පරිණාමය

$y = a + bx$ —— ① යැයි සිතමු.

එවිට, $\bar{y} = a + b\bar{x}$ —— ② වේ.

$\bar{x} = 45$ ද, $\bar{y} = 50$ ද, $x = 70$ ද $y = 80$
ද බව දී ඇත.

② න් $50 = a + 45b$ —— ③

① න් $80 = a + 70b$ —— ④

③ හා ④ විසඳීමෙන් $a = -4$ හා $b = 1.2$ බව ලැබේ.

(i) $\therefore$ රේඛීය පරිණාමය $y = -4 + 1.2x$ වේ.

(ii) විචලතාවයේ අර්ථ දැක්වීමට අනුව,

$$\text{Var } x = \frac{1}{n}\sum_{i=1}^{n}(x_i - \bar{x})^2 \text{ නිසා}$$

$$\text{Var } y = \frac{1}{n}\sum_{i=1}^{n}(y_i - \bar{y})^2$$

$$= \frac{1}{n}\sum_{i=1}^{n}[a + bx_i - (a + b\bar{x})]^2$$

$$= \frac{1}{n}\sum_{i=1}^{n} b^2(x_i - \bar{x})^2$$

$$= b^2 \frac{1}{n}\sum_{i=1}^{n}(x_i - \bar{x})^2$$

$\therefore$ සම්මත අපගමනය $y = b \times$ (සම්මත අපගමනය $x$)

සම්මත අපගමනය $y = 15$ නිසාත් හා $b = 1.2$ නිසාත්,

$$15 = 1.2 \times (\text{ස.අ.}x)$$

$\therefore$ මුල් ලකුණුවල ($x$) සම්මත අපගමනය

$$= \frac{15}{1.2} = 12.5$$

(iii) පරිමාණනය මගින් ලකුණ වෙනස් නොවේ නම් $x = y$ විය යුතු ය.

$\therefore$ ① න්
$$x = a + bx$$
$$x = -4 + 1.2x$$
$$4 = 0.2x$$
$$\underline{x = 20}$$

පරිමාණනය කරන ලද අවමය ලකුණ 2 නම්
① න්

$$2 = -4 + 1.2x_{min} \Rightarrow x_{min} = 5$$

උපරිම ලකුණ 92 නම්,

$$92 = -4 + 1.2_{max} \Rightarrow x_{max} = 80$$

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