ඌව පළාත් අධ්‍යාපන දෙපාර්තමේන්තුව

අධ්‍යයන පොදු සහතික පත්‍ර (උසස් පෙළ) විභාගය 2025

අවසාන වාර පරීක්ෂණය

13 ශ්‍රේණිය

සංයුක්ත ගණිතය 1

ලකුණු ලබා දීමේ පටිපාටිය

① $n=1$ විට

$$L.H.S = \sum_{r=1}^{1} \frac{2^r-1}{2^r} = \frac{2-1}{2} = \frac{1}{2}$$

$$R.H.S = \frac{1}{2} + 1 - 1 = \frac{1}{2}$$ 05

$\therefore$ $n=1$ ට සත්‍ය වේ.

$n=p$ ට සත්‍ය යැයි උපකල්පනය කරමු. $p \in \mathbb{Z}^+$

$$\sum_{r=1}^{p} \frac{2^r-1}{2^r} = \frac{1}{2^p} + p - 1$$ 05

$n=p+1$ විට

$$\sum_{r=1}^{p+1} \frac{2^r-1}{2^r} = \sum_{r=1}^{p} \frac{2^r-1}{2^r} + \frac{2^{p+1}-1}{2^{p+1}}$$ 05

$$= \frac{1}{2^p} + p - 1 + \frac{2^{p+1}-1}{2^{p+1}}$$

$$= \frac{2 + 2^{p+1} - 1}{2^{p+1}} + p - 1$$ 05

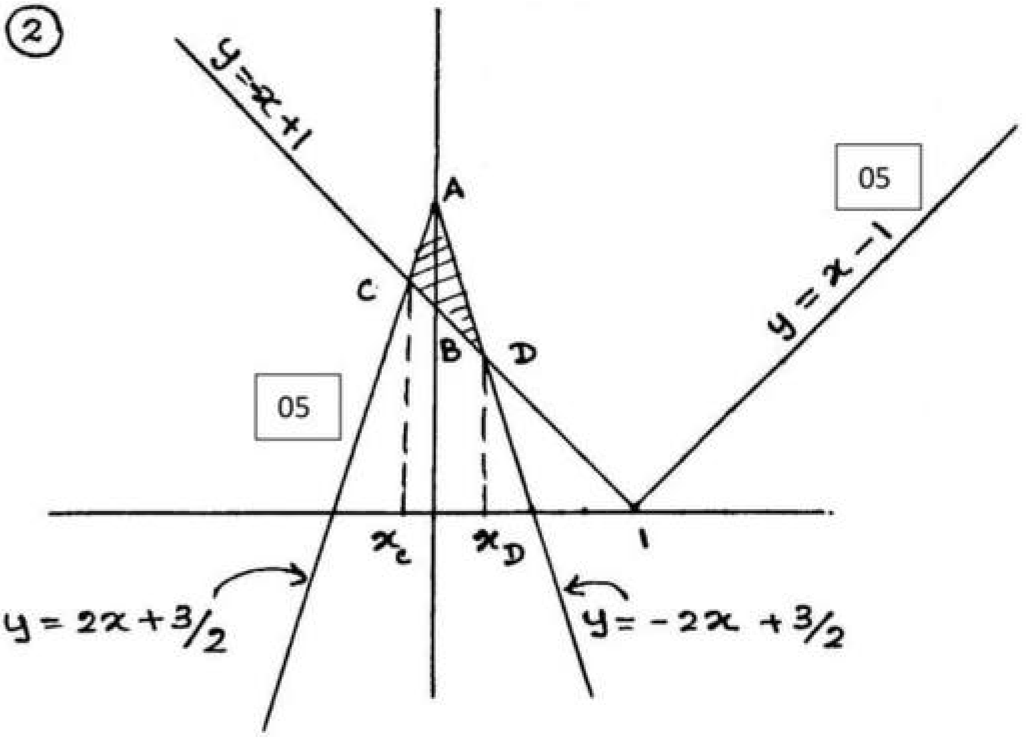
$$= \frac{1}{2^{p+1}} + \overline{p+1} - 1$$ 05

$\therefore$ $n=p+1$ ට සත්‍ය වේ. 05

$\therefore$ ගණිත අභ්‍යුහන මූලධර්මය අනුව සියලු $n \in \mathbb{Z}^+$ සඳහා සත්‍ය වේ.

25

②



$x_D /\quad -x+1 = -2x+\frac{3}{2} \Rightarrow x_D = \frac{1}{2}$

$x_c /\quad -x+1 = 2x+\frac{3}{2} \Rightarrow x_c = -\frac{1}{6}$ $\boxed{05}$

$ACB\ \Delta = \frac{1}{2}\times\frac{1}{2}\times\frac{1}{6} = \frac{1}{24}$

$\boxed{05}$

$ABD\ \Delta = \frac{1}{2}\times\frac{1}{2}\times\frac{1}{2} = \frac{1}{8}$

$\therefore\ \Delta$ යේ වර්ගඵලය $= \frac{1+3}{24} = \frac{1}{6}$ වර්ග ඒකක. $\boxed{05}$

$\boxed{25}$

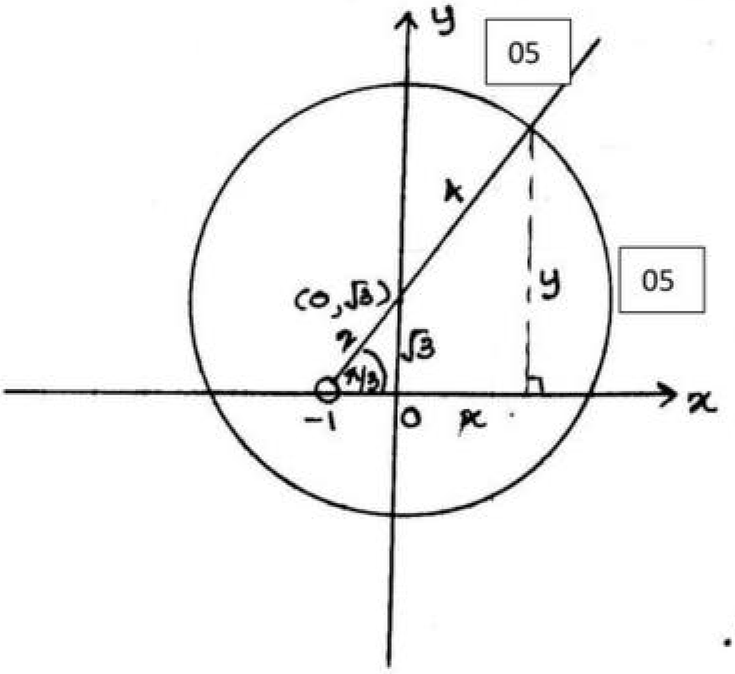
③ $|iz+\sqrt{3}| = 4$

$|z-\sqrt{3}i| = 4$

$|z-(0+\sqrt{3}i)| = 4$

$\text{Arg}\,(z+1) = \pi/3$

$\text{Arg}\,[z-(-1+0i)] = \pi/3$



$\boxed{05}$ $\boxed{05}$

$\tan \pi/3 = \frac{y}{1+x}$

$y = \sqrt{3}(x+1)$ $\boxed{05}$

$(1+x)^2+y^2 = 36$

$\left(\frac{y}{\sqrt{3}}\right)^2+y^2 = 36$

$y^2 = 27$

$y = 3\sqrt{3}$ $\boxed{05}$

$x = 2$

$\therefore\ z = 2+3\sqrt{3}\,i$ $\boxed{05}$

$\boxed{25}$

④ $(2-x)^6 = \sum_{r=0}^{6} {}^6C_r\, 2^{6-r}(-x)^r$

$T_1 = {}^6C_0\, 2^6(-x)^0 = {}^6C_0\, 2^6 = 64$ $\boxed{05}$

$T_2 = {}^6C_1\, 2^{6-1}(-x)^1 = {}^6C_1\, 2^5(-x) = -192x$ $\boxed{05}$

$T_3 = {}^6C_2\, 2^{6-2}(-x)^2 = 15.2^4 x^2 = 240x^2$ $\boxed{05}$

$(1+kx)(2-x)^6 = (1+kx)(64-192x+240x^2+\dots)$

$x^2 \Rightarrow 240-192k = 0$ $\boxed{05}$

$k = \frac{240}{192} = \frac{20}{16} = \underline{\underline{\frac{5}{4}}}$ $\boxed{05}$

$\boxed{25}$

⑤ $\lim_{x \to 9} \frac{\sqrt{x-8}-1}{\sin[\pi(\sqrt{x}-3)]}$

$= \lim_{x \to 9} \frac{\pi(\sqrt{x}-3)}{\sin \pi(\sqrt{x}-3)} \times \frac{(\sqrt{x-8}-1)(\sqrt{x-8}+1)}{(\sqrt{x-8}+1)\,\pi(\sqrt{x}-3)}$ $\boxed{05}$ $\boxed{05}$

$= \frac{1}{2\pi} \times \lim_{x \to 9} \frac{x-8-1}{x^{1/2}-9^{1/2}}$ $\boxed{05}$ $\boxed{05}$

$= \frac{1}{2\pi} \quad x \longrightarrow 9 \quad \frac{x-9}{x^{1/2}-9^{1/2}}$

$= \frac{1}{2\pi} \quad \frac{1}{\frac{1}{2}\times(3^2)^{-1/2}}$ $\boxed{05}$

$= \frac{3}{\pi}$

$\boxed{25}$

⑥ $y = \sqrt{\ln x}$

$$V = \int_2^4 \pi y^2 \, dx \quad \boxed{05}$$

$$= \pi \int_2^4 \ln x \, dx.$$

$$= \pi \int_2^4 \ln x \frac{d}{dx} x \, dx \quad \boxed{05}$$

$$= \pi \left\{ \left[x \ln x \right]_2^4 - \int_2^4 1 \, dx \right\} \quad \boxed{05}$$

$$= \pi \left\{ 4 \ln 4 - 2 \ln 2 - 4 + 2 \right\} \quad \boxed{05}$$

$$= \pi \left\{ \ln \frac{4^4}{4} - 2 \right\}$$

$$= \pi \left[\ln 4^3 - 2 \right\} \quad \boxed{05}$$

$$= 2\pi \left[3 \ln 2 - 1 \right]$$

$\boxed{25}$

⑦ $x = 8 \cos t$, $y = 4 \sin 2t$; $0 < t < \pi/2$

$$8 \cos t = 4\sqrt{3} \qquad 4 \sin 2 \frac{\pi}{6} = 4 \frac{\sqrt{3}}{2} = 2\sqrt{3}$$

$$\cos t = \frac{\sqrt{3}}{2}$$

$$t = \frac{\pi}{6}. \quad \boxed{05}$$

$$\frac{dx}{dt} = -8 \sin t \quad \boxed{05} \qquad \frac{dy}{dx} = \frac{-\cos 2t}{\sin t} \quad \boxed{05}$$

$$\frac{dy}{dt} = 8 \cos 2t \qquad \left(\frac{dy}{dx} \right)_{t = \pi/6} = \frac{-\cos 2 \cdot \pi/6}{\sin \pi/6}.$$

$$= -\frac{1}{2} \times \frac{2}{1}$$

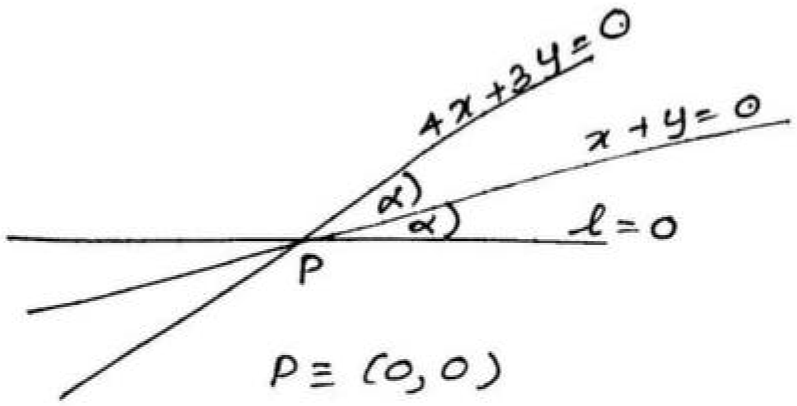
$$= -1 \quad \boxed{05}$$

$$\therefore l: \; y - 2\sqrt{3} = +1 (x - 4\sqrt{3})$$

$$x - y - 2\sqrt{3} = 0 \quad \boxed{05}$$

$\boxed{25}$

⑧



$P \equiv (0,0)$

$$\tan\alpha = \left|\frac{-\frac{4}{3}+1}{1+(-\frac{4}{3})(-1)}\right|$$

$$= \frac{1}{7} \quad \boxed{05}$$

$l:\ y=mx$ ගනිමු.

$$\frac{1}{7} = \left|\frac{m+1}{1-m}\right| \quad \boxed{05}$$

$\pm(1-m) = 7m+7$

$(+) \Rightarrow 8m = -6$ $\quad (-)\ 6m = -8$

$m = -3/4$ $\quad \boxed{05}$ $\quad m = -4/3$

$\therefore m = -3/4$ $\quad \therefore l:\ y = -\frac{3}{4}x.$

$3x+4y=0$ $\quad \boxed{05}$

$l + \lambda l_1 = 0$

$\lambda(x+7y-1) + \mu(3x+4y) = 0$

$(0,0) \Rightarrow \lambda = 0$

$\therefore$ රේඛාව $3x+4y=0$ $\boxed{05}$ $\quad \boxed{25}$

⑨ $x = 3\cos\theta \quad , \quad y = -2 + 3\sin\theta$

$$x^2 + y^2 + 4y - 5 = 9\cos^2\theta + 4 + 9\sin^2\theta - 12\sin\theta + 4(-2 + 3\sin\theta) - 5$$
$$= 9(\cos^2\theta + \sin^2\theta) + 4 - 8 - 5$$
$$= 9 + 4 - 13$$
$$= 0 \quad \boxed{05}$$

$\therefore$ $x = 3\cos\theta$, $y = -2 + 3\sin\theta$ මගින් $S = 0$ නිරූපණය කරයි.

$\theta = \pi/2$ විට $P \equiv (0, 1)$ $\boxed{05}$

$\therefore$ ස්පර්ශකය : $x.0 + y.1 + 0(x+0) + 2(y+1) - 5 = 0$ $\boxed{05}$

$$y + 2y + 2 - 5 = 0$$
$$3y = 3$$
$$y = 1 \quad \boxed{05}$$

පොදු ජ්‍යාය:

$$(x^2 + y^2 + 4y - 5) - (x^2 + y^2 - 2x) = 0$$

$$4y + 2x - 5 = 0 \quad \boxed{05}$$

$\boxed{25}$

⑩ $$\sin\theta = \frac{2\sin\theta/2 \cos\theta/2}{\cos^2\theta/2 + \sin^2\theta/2} \quad \boxed{05}$$

$$= \frac{2\tan\theta/2}{1 + \tan^2\theta/2} = \frac{2t}{1+t^2}$$

$\theta = \pi/4$ විට $t = \tan\pi/8$ $\boxed{05}$

$$\sin\pi/4 = \frac{2t}{1+t^2}$$

$$\frac{1}{\sqrt{2}} = \frac{2t}{1+t^2}$$

$$t^2 - 2\sqrt{2}t + 1 = 0 \quad \boxed{05}$$

$$t = \frac{2\sqrt{2} \pm \sqrt{8-4}}{2}$$

$$t = \frac{2\sqrt{2} \pm 2}{2}$$
$$= \sqrt{2} \pm 1 \quad \boxed{05}$$

$$\tan\pi/8 < 1$$

$$\therefore \tan\pi/8 = \sqrt{2} - 1 \quad \boxed{05}$$

$\boxed{25}$

⑪ a) $f(x) = (k-1)x^2 - 2x + k - 1 \quad ; \quad k \neq 1$

$\Delta > 0$ $\boxed{05}$

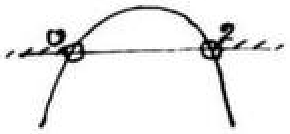
$(-2)^2 - 4(k-1)(k-1) > 0$ $\boxed{05}$

$4 - 4(k-1)^2 > 0$

$1 - (k-1)^2 > 0$

$(1-k+1)(1+k-1) > 0$

$k(2-k) > 0$ $\boxed{05}$



$\boxed{05}$ $k < 0$ හෝ $k > 2$ ——①

$\alpha + \beta = \dfrac{2}{k-1}$ $\boxed{05}$ $\boxed{05}$ $\qquad \alpha + \beta > 0$.

$\Rightarrow \dfrac{2}{k-1} > 0$ $\boxed{05}$

$\alpha\beta = \dfrac{k-1}{k-1} = 1$ $\boxed{05}$ $\quad \Rightarrow k > 1$ $\boxed{05}$ ——②

$\boxed{25}$

① හා ② න් $k > 2$. $\boxed{05}$

$\boxed{25}$

$f(1) = (k-1) - 2 + k - 1$ $\boxed{05}$

$= 2k - 4$ $\boxed{05}$

$k > 2 \Rightarrow 2k - 4 > 0$ $\boxed{05}$

$\therefore f(1) \neq 0$ $\boxed{05}$

$\therefore$ 1, $f(x)$ හි මූලයක් නොවේ.

$\boxed{20}$

$f(x) = 0$ හි මූල α හා β නම්

$x = \alpha, \quad x = \beta$

$\dfrac{1}{\alpha - 1} = y$ හා $\dfrac{1}{\beta - 1} = y$ නම්

$y = \dfrac{1}{x-1} \Rightarrow x - 1 = \dfrac{1}{y}$

$x = \dfrac{1+y}{y}$ $\boxed{05}$

$\therefore \dfrac{1}{\alpha-1}$ හා $\dfrac{1}{\beta-1}$ මූල වන සමී.

$f(x) = 0$ හි ආදේශ.

$(k-1)\dfrac{(1+y)^2}{y^2} - 2\left(\dfrac{1+y}{y}\right) + k - 1 = 0$ $\boxed{05}$

$(k-1)(1+2y+y^2) - 2(1+y)y + (k-1)y^2 = 0$

$2(k-1)y^2 - 2y^2 + 2(k-1)y - 2y + (k-1) = 0$

$2(k-2)y^2 + 2(k-2)y + k - 1 = 0$ $\boxed{05}$

$\boxed{15}$

b) $f(x) = (x-1)Q(x) + a.$

$f(x) = (x-2)Q'(x) + b.$

$f(1) = a \ \boxed{05} \quad f(2) = b. \ \boxed{05}$

$f(x) = (x-1)(x-2)g(x) + Ax + B$ ගනිමු.

$f(1) = A + B \Rightarrow A + B = a$ —— ① $\boxed{05}$

$f(2) = 2A + B \Rightarrow 2A + B = b.$ —— ② $\boxed{05}$

② − ① $A = b - a \ \boxed{05}$

$\therefore B = a - b + a$

$= 2a - b. \ \boxed{05}$

$\therefore Ax + B = (b-a)x + 2a - b.$

$= (b-a)(x-1) + b - a + 2a - b.$

$= (b-a)(x-1) + a \ \boxed{05}$

$\therefore \lambda = (b-a), \ \mu = a \ \boxed{05}$

$\therefore f(x) = (x-1)(x-2)g(x) + (b-a)(x-1) + a \ \boxed{05}$

$x^5 + kx^2 = (x-1)(x-2)g(x) + (b-a)(x-1) + a$

$x = 1$ ආදේශ

$1 + k = a$

$k = a - 1 \longrightarrow (1) \ \boxed{05}$

x=2 $\Longrightarrow 4k = b - 32 \longrightarrow (2) \ \boxed{05}$

$x^0 \Longrightarrow 0 = 2a - b \longrightarrow (3) \ \boxed{05}$

(1), (2), (3) $\Longrightarrow k = -15 \ \boxed{05}$

$\boxed{65}$

⑫ a) i) $^2C_1 \times {}^5C_3 \times {}^4C_2$ $\boxed{05}$

$$= \frac{\cancel{2!}}{1!\,1!} \times \frac{5!}{\cancel{2!}\,3!} \times \frac{4!}{2!\,2!} \quad \boxed{05}$$

$$= \frac{5 \times 4 \times 4 \times 3}{2}$$

$$= 120 \quad \boxed{05}$$

$\boxed{15}$

ii) :-

T	B	G
1	1+1	1+2
1	1+2	1+1
2	1+1	1+1

$$^2C_2 \times {}^4C_1 \times {}^3C_1 + {}^2C_1 \times {}^4C_1 \times {}^3C_2 + {}^2C_1 \times {}^4C_2 \times {}^3C_1 \quad \boxed{05 \times 3}$$

$$= 2\left[\frac{4!}{3!\,1!} \times \frac{3!}{2!\,1!} + \frac{4!}{2!\,2!} \times \frac{3!}{2!\,1!}\right] + 12 \quad \boxed{10}$$

$$= 2\left[4 \times 3 + \frac{4 \times 3}{2} \times 3\right]$$

$$= 2\,(12 + 18) + 12$$

$$= 72 \quad \boxed{10}$$

$\boxed{35}$

b) $n \in \mathbb{Z}^+$ $\displaystyle\sum_{r=1}^{n} U_r = \frac{1}{2} - \frac{1}{2(n^2+n+1)}$ —— ①

$$\sum_{r=1}^{n-1} U_r = \frac{1}{2} - \frac{1}{2[(n-1)^2 + (n-1) + 1]} \quad ;\ n \geq 2 \quad \boxed{05}$$

$$= \frac{1}{2} - \frac{1}{2[n^2 - 2n + 1 + n - 1 + 1]}$$

$$= \frac{1}{2} - \frac{1}{2(n^2 - n + 1)} \quad \text{—— ②} \quad \boxed{05}$$

① - ②

$$U_n = \frac{n^2 + n + 1 - n^2 + n - 1}{2(n^2+n+1)(n^2-n+1)} \quad \boxed{05}$$

$$= \frac{n}{(n^2+1-n)(n^2+1+n)}$$

$$=$$

$$U_n = \frac{n}{(n^2+1)^2 - n^2}$$

$$= \frac{n}{n^4+2n^2+1-n^2}$$

$$= \frac{n}{n^4+n^2+1} \quad \boxed{05}$$

$n=1$ විට

$$\sum_{r=1}^{1} U_r = U_1 = \frac{1}{2} - \frac{1}{2(3)} = \frac{1}{3} \quad \boxed{05}$$

$$U_1 = \frac{1}{1+1+1} = \frac{1}{3} \quad \boxed{05}$$

∴ සියලු $n \in \mathbb{Z}^+$ සඳහා

$$U_n = \frac{n}{n^4+n^2+1} \text{ වේ.} \quad \boxed{05}$$

$\boxed{35}$

$$2U_r = \frac{A}{r^2-r+1} - \frac{A}{r^2+r+1}$$

$$= \frac{A}{(r^2+1)-r} - \frac{A}{(r^2+1)+r.} \quad \boxed{05}$$

$$\frac{2r}{r^4+r^2+1} = \frac{A[r^2+r+1] - A[r^2-r+1]}{(r^2+1)^2 - r^2} \quad \boxed{05}$$

$$\frac{2r}{r^4+r^2+1} = \frac{2Ar}{r^4+r^2+1}$$

$$\Rightarrow \quad 2A = 2 \Rightarrow A = 1 \quad \boxed{05}$$

$$\therefore \; V_r = \frac{-1}{r^2+r+1} \text{ නම් } V_{r-1} = \frac{-1}{(r-1)^2+(r-1)+1}$$

$$= \frac{-1}{r^2-r+1} \quad \boxed{05}$$

(10)

$$\therefore\ 2U_r = V_r - V_{r-1}\ ;\ V_r = \frac{-1}{r^2+r+1}$$ $\boxed{25}$

$\boxed{05}$

$$\begin{aligned}
2U_1 &= \cancel{V_1} - V_0 \\
2U_2 &= V_2 - \cancel{V_1} \quad \boxed{05} \\
&\ \vdots \qquad \vdots \\
2U_{n-1} &= \cancel{V_{n-1}} - V_{n-2} \\
2U_n &= V_n - \cancel{V_{n-1}} \quad \boxed{05}
\end{aligned}$$

$$2\sum_{r=1}^{n} U_r = -V_0 + V_n$$

$$= 1 + \frac{1}{n^2+n+1} \quad \boxed{05}$$

$$\sum_{r=1}^{n} U_r = \frac{1}{2} + \frac{1}{2(n^2+n+1)} \quad \boxed{05}$$ $\boxed{20}$

$$\sum_{r=1}^{\infty} U_r = \lim_{n\to\infty} \frac{1}{2} + \frac{1}{2(n^2+n+1)} \quad \boxed{05}$$

$$= \frac{1}{2}$$ (පරිමිත වේ) $\boxed{05}$

$\therefore$ ශ්‍රේණිය අභිසාරී වේ. $\boxed{05}$

$$\sum_{r=1}^{\infty} U_r = \frac{1}{2} \quad \boxed{05}$$ $\boxed{20}$

⑬

$$A^T = \begin{pmatrix} 5 & p & -2 \\ -1 & 0 & b \end{pmatrix}$$

$$A^T B = C$$

$$\begin{pmatrix} 5 & p & -2 \\ -1 & 0 & q \end{pmatrix}\begin{pmatrix} 0 & 3 \\ 1 & 0 \\ 0 & 1 \end{pmatrix} = \begin{pmatrix} 2 & 13 \\ 0 & -2 \end{pmatrix}$$

$\boxed{05}$

$\boxed{10}$ $$\begin{pmatrix} p & 13 \\ 0 & q-3 \end{pmatrix} = \begin{pmatrix} 2 & 13 \\ 0 & -2 \end{pmatrix}$$

$$p = 2 \qquad q - 3 = -2$$

$$p = 2 \;, q = 1 \quad \boxed{05}$$

$\boxed{20}$

$$\det C = -4 - 0 = -4 \neq 0 \quad \boxed{05}$$

$\therefore C^{-1}$ පවතී $\boxed{05}$

$\boxed{10}$

$$C^{-1} = \frac{1}{\det C}\begin{pmatrix} -2 & -13 \\ 0 & 2 \end{pmatrix}$$

$$= \frac{1}{-4}\begin{pmatrix} -2 & -13 \\ 0 & 2 \end{pmatrix} \quad \boxed{05}$$

$$= \begin{pmatrix} +\frac{1}{2} & \frac{13}{4} \\ 0 & \frac{1}{2} \end{pmatrix} \quad \boxed{05}$$

$\boxed{10}$

$$DC = \begin{pmatrix} 0 & 2 \\ 2 & 11 \end{pmatrix}$$

$$DCC^{-1} = \begin{pmatrix} 0 & 2 \\ 2 & 11 \end{pmatrix}\begin{pmatrix} 1/2 & 13/4 \\ 0 & 1/2 \end{pmatrix} \quad \boxed{05}$$

$\boxed{05}$

$$D = \begin{pmatrix} 0 & 1 \\ 1 & 12 \end{pmatrix} \quad \boxed{05}$$

$\boxed{15}$

b) $z_1 = x_1 + iy_1 \qquad z_2 = x_2 + iy_2$

$$\frac{z_1}{z_2} = \frac{(x_1+iy_1)}{(x_2+iy_2)} \times \frac{(x_2-iy_2)}{(x_2-iy_2)}$$

$$= \frac{x_1x_2 + y_1y_2 + i(y_1x_2 - x_1y_2)}{x_2^2 + y_2^2} \quad \boxed{05}$$

$$\left|\frac{z_1}{z_2}\right| = \sqrt{\frac{(x_1x_2+y_1y_2)^2 + (y_1x_2 - x_1y_2)^2}{(x_2^2+y_2^2)^2}} \quad \boxed{05}$$

$$= \frac{\sqrt{x_1^2x_2^2 + 2x_1x_2y_1y_2 + y_1^2y_2^2 + y_1^2x_2^2 - 2y_1x_2x_1y_2 + x_1^2y_2^2}}{\sqrt{x_2^2+y_2^2}}$$

$$= \frac{\sqrt{x_1^2(x_2^2+y_2^2) + y_1^2(x_2^2+y_2^2)}}{\sqrt{x_2^2+y_2^2}}$$

$$= \frac{\sqrt{x_1^2+y_1^2}}{\sqrt{x_2^2+y_2^2}} \quad \boxed{05}$$

$$= \frac{|z_1|}{|z_2|} \quad \boxed{05}$$

$\boxed{20}$

$$\therefore \left|\frac{z_1}{z_2}\right| = \frac{|z_1|}{|z_2|}$$

$$\left|\frac{z-5i}{z+5i}\right| = 1$$

$$|z-5i| = |z+5i| \quad \boxed{05}$$

$$|x+iy-5i| = |x+iy+5i|$$

$$|x+i(y-5)| = |x+i(y+5)|$$

$$\sqrt{x^2+(y-5)^2} = \sqrt{x^2+(y+5)^2} \quad \boxed{05}$$

$$x^2+(y-5)^2 = x^2+(y+5)^2$$

$$x^2+y^2-10y+25 = x^2+y^2+10y+25$$

$\boxed{15}$

$$y = 0 \quad \boxed{05}$$

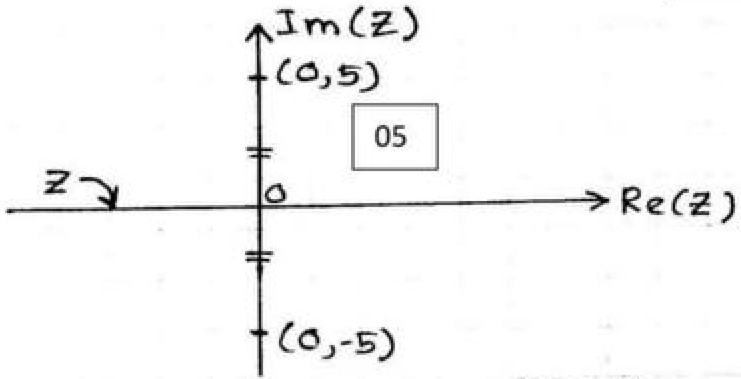
$\therefore z$ සංකීර්ණ සංඛ්‍යාව තාත්වික අක්ෂය මත පිහිටයි. (13)

Aliter

$$\left|\frac{z-5i}{z+5i}\right| = 1$$

$|z-5i| = |z+5i|$ 05

$|z-(0+5i)| = |z-(0-5i)|$ 05



05

15

z, Re(z) අක්ෂය මත පිහිටයි.

ii) $z = x+iy$ ගනිමු.

$\bar{z} = x-iy$ 05

$z\bar{z} = (x+iy)(x-iy)$

$= x^2+y^2$ 05

$z\bar{z} = |z|^2$

10

$z+\bar{z} = x+iy+x-iy$

$= 2x$ 05

$= 2\,\mathrm{Re}(z)$

05

$|z_1+z_2|^2 = (z_1+z_2)(\overline{z_1+z_2})$ 05

$= (z_1+z_2)(\bar{z}_1+\bar{z}_2)$

$= z_1\bar{z}_1 + z_1\bar{z}_2 + z_2\bar{z}_1 + z_2\bar{z}_2$ 05

$= |z_1|^2 + z_1\bar{z}_2 + z_2\bar{z}_1 + |z_2|^2$ 05

$|z_1+z_2|^2 = |z_1|^2 + |z_2|^2$ නිසා

$|z_1|^2+|z_2|^2 = |z_1|^2 + 2\,\mathrm{Re}\, z_1\bar{z}_2 + |z_2|^2$

$\mathrm{Re}\, z_1\bar{z}_2 = 0$ 05

$\therefore z_1\bar{z}_2$ ශුද්ධ අතාත්වික වේ. 05

25

$$z = \frac{1}{2} + \frac{\sqrt{3}}{2}i$$

$$z = \cos\frac{\pi}{3} + i\sin\frac{\pi}{3}$$

$$z^8 = \left(\cos\frac{\pi}{3} + i\sin\frac{\pi}{3}\right) \quad \boxed{05}$$

ද මුවාවර් ප්‍රමේයයෙන්

$$z^8 = \cos\frac{8\pi}{3} + i\sin\frac{8\pi}{3}$$

$$= \cos\left(3\pi - \frac{\pi}{3}\right) + i\sin\left(3\pi - \frac{\pi}{3}\right)$$

$$= -\cos\pi/3 + i\sin\pi/3$$

$$z^5 = \left(\cos\pi/3 + i\sin\pi/3\right)^5 \quad \text{—①} \quad \boxed{05}$$

ද මුවාවර් ප්‍රමේයයෙන්

$$z^5 = \cos\frac{5\pi}{3} + i\sin\frac{5\pi}{3}$$

$$= \cos(2\pi - \pi/3) + i\sin(2\pi - \pi/3)$$

$$= \cos\pi/3 - i\sin\pi/3 \quad \text{—②} \quad \boxed{05}$$

① + ② න්

$$\boxed{20}$$

$$z^8 + z^5 = 0 \quad \boxed{05}$$

14) $x \neq -1$ $\qquad f(x) = \dfrac{3x-1}{(x+1)^2}$

$\boxed{05}$ $\qquad \boxed{05}$

$$f'(x) = \frac{(x+1)^{\cancel{2}} . 3 \quad - (3x-1)\, 2 \cancel{(x+1)}}{(x+1)^{\cancel{4}\,3}}$$

$\boxed{05}$

$$= \frac{3x+3-6x+2}{(x+1)^3}$$

$\boxed{05}$

$$= \frac{-3x+5}{(x+1)^3}$$

$\boxed{20}$

$f'(x) = 0 \Rightarrow \quad x = 5/3$ $\boxed{05}$ $\quad f(5/3) = \dfrac{4 \times 9}{64} = \dfrac{9}{16}$

$(5/3, 9/16)$ හැරුම ලක්ෂ්‍යයකි.

	$x < -1$	$-1 < x < 5/3$	$x > 5/3$
$f'(x)$	$(-)$	$(+)$	$(-)$
x සමග $f(x)$	අඩුවේ	වැඩිවේ	අඩුවේ.

$\boxed{15}$

$(5/3, 9/16)$ ස්ථානීය උපරිමයකි. $\boxed{05}$

$\boxed{25}$

$f''(x) = \dfrac{6(x-3)}{(x+1)^4}$ $\qquad f''(x) = 0 \Rightarrow x = 3$ $\boxed{05}$

	$x < -1$	$-1 < x < 3$	$x > 3$
$f''(x)$	$(-)$	$(-)$	$(+)$
අවතලතාව	යටි	යටි	උඩු

$\boxed{10}$

$f(3) = 8/16 \qquad \therefore \left(3, \dfrac{8}{16}\right)$ $\boxed{05}$ නතිවර්තන ලක්ෂ්‍යයකි.

$\boxed{20}$

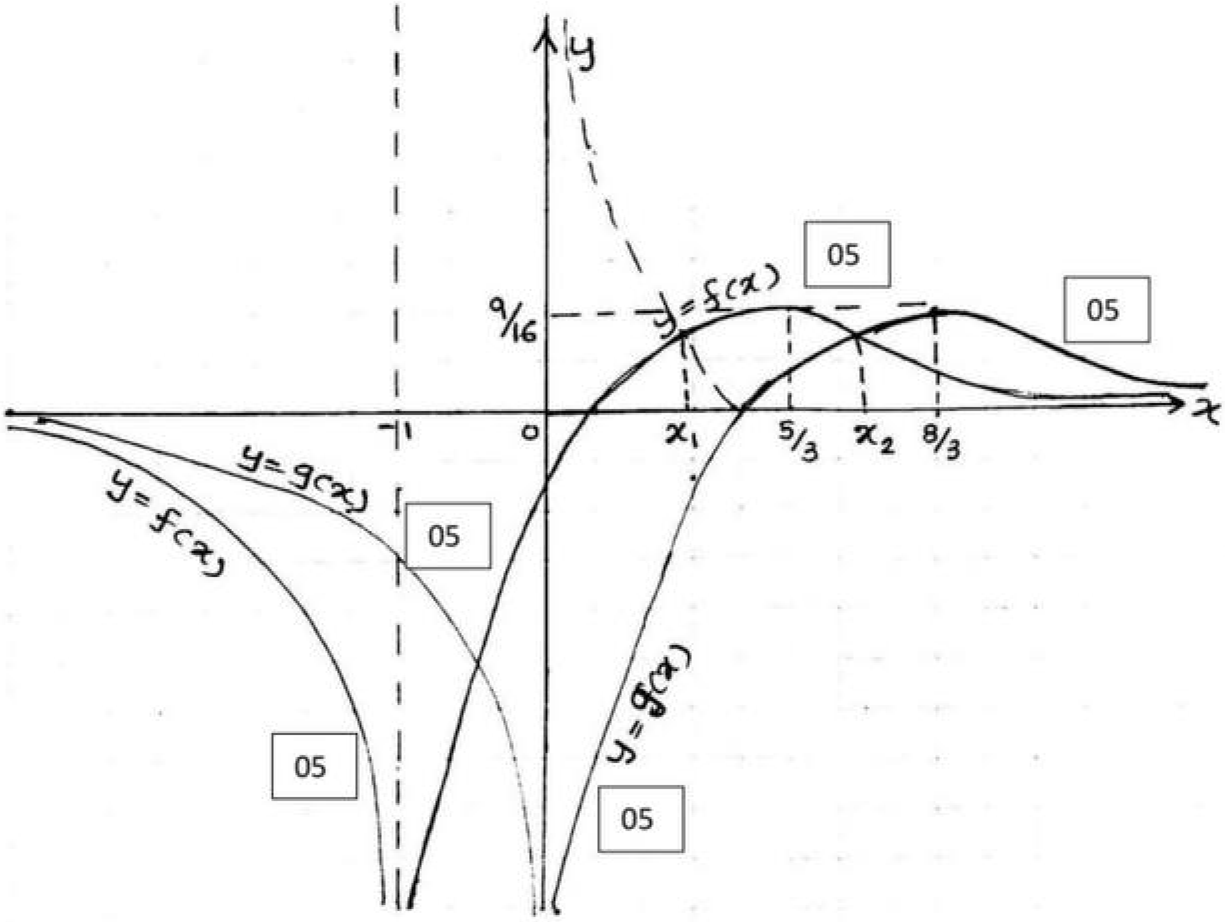
$x \longrightarrow -1^-$ විට $f(x) \rightarrow -\infty$ $\qquad \therefore x = -1$ සිරස්

$x \longrightarrow -1^+$ විට $f(x) \rightarrow -\infty$ $\qquad$ ස්පර්ශෝන්මුඛයකි. $\boxed{05}$

$\boxed{05}$

$\lim\limits_{x \to \pm\infty} f(x) = 0 \qquad \therefore y = 0$ තිරස් ස්පර්ශෝන්මුඛයකි.

(16)



35

$$g(x) = f(x-1) = \frac{3(x-1)-1}{(x-1+1)^2} = \frac{3x-4}{(x)^2} = g(x)$$

05

∴ $y=f(x)$ හි ප්‍රස්තාරය x අක්ෂයේ ධන දිශාව ඔස්සේ ඒකක 1 ක් විස්ථාපනයෙන් $y=g(x)$ හි ප්‍රස්තාරය ලැබේ.

$0<x<8/3$ තුළ $f(x)=|g(x)|$ සඳහා විසඳුම 2 ක් පවතී (x_1, x_2) 05

10

b) $V = \pi R^2 H - \frac{2}{3}\pi\left(\frac{R}{2}\right)^3$ 05

$$= \pi R^2 H - \frac{\pi R^3}{12}$$

$$\left(\frac{12V}{\pi R^2} + R\right)\frac{1}{12} = H$$

$$S = 2\pi RH + 2\pi R^2 - \pi\left(\frac{R}{2}\right)^2 + 2\pi\left(\frac{R}{2}\right)^2 \quad \boxed{05}$$

$$= 2\pi R\left(\frac{V}{\pi R^2} + \frac{R}{12}\right) + \frac{(8 - 1 + 2)\pi R^2}{4}$$

$$= \frac{2V}{R} + \frac{\pi R^2}{6} + \frac{9}{4}\pi R^2 \quad \boxed{05}$$

$$= \frac{2V}{R} + \frac{29}{12}\pi R^2$$

$$\frac{dS}{dR} = \frac{-2V}{R^2} + \frac{29}{12}\pi 2R$$

$$= \frac{-2V}{R^2} + \frac{29\pi R}{6} \quad \boxed{05}$$

$$\frac{dS}{dR} = 0 \; \boxed{05} \iff \frac{29\pi R}{6} = \frac{2}{R^2} \quad 4176\pi$$

$$R^3 = \frac{4176 \times 12}{29} = 12^3$$

$$R = 12\,\text{cm} \quad \boxed{05}$$

$0 < R < 12$ විට $\frac{dS}{dR} < 0$ $\boxed{05}$ $\therefore R = 12$ විට S අවම වේ. $\boxed{05}$

$R > 12$ විට $\frac{dS}{dR} > 0$

$\boxed{40}$

(15)
$$1 = A(x^2+1)(x+1) + (Bx+C)(x-2)(x+1) + D(x-2)(x^2+1) \quad — ①$$

$$x=-1 \Rightarrow \quad 1 = C(-3)(2)$$
$$C = -\tfrac{1}{6} \quad \boxed{05}$$

$$x=2 \Rightarrow \quad 1 = A(5)(3)$$
$$A = \tfrac{1}{15} \quad \boxed{05}$$

$$x=0 \Rightarrow \quad 1 = A + C(-2) + D(-2)$$
$$1 = \frac{1}{15} + \frac{1}{3} - 2D$$
$$2D = \frac{6}{15} - 1$$
$$D = \frac{-9}{15} \times \frac{1}{2} = \frac{-3}{10} \quad \boxed{05}$$

$$x^3 \Rightarrow \quad 0 = A + B + D$$
$$B = -(A+D)$$
$$= -\left(\frac{1}{15} - \frac{3}{10}\right) = \frac{-(2-9)}{30}$$
$$= \frac{7}{30} \quad \boxed{05}$$

$\boxed{20}$

$$① \div (x^2+1)(x+1)(x-2)$$

$$\frac{1}{(x^2+1)(x+1)(x-2)} = \frac{A}{x-2} + \frac{Bx+C}{x^2+1} + \frac{D}{x+1}$$
$$= \frac{1}{15(x-2)} + \frac{\frac{7}{30}x - \frac{1}{6}}{x^2+1} - \frac{3}{10(x+1)} \quad \boxed{35}$$
$$= \frac{1}{15(x-2)} + \frac{7x-5}{30(x^2+1)} - \frac{3}{10(x+1)} \quad \boxed{05}$$

$$\int \frac{1}{(x-2)(x^2+1)(x+1)}\,dx = \frac{1}{15}\int \frac{1}{x-2}\,dx + \frac{1}{30}\int \frac{7x-5}{x^2+1}\,dx - \frac{3}{10}\int \frac{1}{x+1}\,dx. \quad \boxed{05}$$

$$= \frac{1}{15}\ln|x-2| + \frac{7}{60}\ln|x^2+1| - \frac{1}{6}\tan^{-1}x - \frac{3}{10}\ln|x+1| + C$$

$\boxed{05x5=25}$

b) $\sin 3x = 3\sin x - 4\sin^2 x \cdot \sin x$

$= 3\sin x - 4(1-\cos^2 x)\sin x$

$= -\sin x + 4\cos^2 x \sin x$ $\boxed{05}$

$= (4\cos^2 x - 1)\sin x.$

$\therefore \int \frac{\sin 3x}{1+\cos^2 x}dx = \int \frac{(4\cos^2 x - 1)\sin x}{1+\cos^2 x} dx$ $\boxed{05}$

$\therefore \int_{\pi/6}^{\pi/3} \frac{\sin 3x}{1+\cos^2 x}dx = \int_{\pi/6}^{\pi/3} \frac{(4\cos^2 x - 1)\sin x}{(1+\cos^2 x)} dx = I$

$t = \cos x$ ආදේ $\qquad x = \pi/6 \Rightarrow t = \sqrt{3}/2$

$dt = -\sin x\, dx$ $\boxed{05}$ $\qquad x = \pi/3 \Rightarrow t = 1/2$ $\boxed{05}$

$I = \int_{\sqrt{3}/2}^{1/2} \frac{4t^2-1}{1+t^2}(-dt)$ $\boxed{05}$

$= -\int_{\sqrt{3}/2}^{1/2} \frac{4(t^2+1)-5}{1+t^2} dt$

$\boxed{05}$ $\boxed{05}$

$= -4\int_{\sqrt{3}/2}^{1/2} dt + 5\int_{\sqrt{3}/2}^{1/2} \frac{1}{1+t^2}dt$

$= -4\left[t\right]_{\sqrt{3}/2}^{1/2}$ $\boxed{05}$ $+ 5\left[\tan^{-1} t\right]_{\sqrt{3}/2}^{1/2}$ $\boxed{05}$ $\boxed{55}$

$= -4\left[\frac{1}{2} - \frac{\sqrt{3}}{2}\right] + 5\left[\tan^{-1}1/2 - \tan^{-1}\sqrt{3}/2\right]$ $\boxed{05}$

$= 2(\sqrt{3}-1) + 5\left[\tan^{-1}1/2 - \tan^{-1}\sqrt{3}/2\right]$ $\boxed{05}$ (20)

c) $\int_0^{\pi/2} f(x)\,dx$ හි $x = \pi/2 - t$ ආදේශයෙන්, $\boxed{05}$

$$dx = -dt$$

$$x = 0 \Rightarrow t = \pi/2$$

$$x = \pi/2 \Rightarrow t = 0 \quad \boxed{05}$$

$$\int_0^{\pi/2} f(x)\,dx = -\int_{\pi/2}^{0} f(\pi/2 - t)\,dt$$

$$= \int_0^{\pi/2} f(\pi/2 - t)\,dt$$

$$= \int_0^{\pi/2} f(\pi/2 - x)\,dx. \quad \boxed{05} \qquad \boxed{15}$$

$$I = \int_0^{\pi/2} \frac{\sin^2 x}{1+\sin^2 x}\,dx = \int_0^{\pi/2} \frac{\sin^2(\pi/2 - x)}{1+\sin^2(\pi/2 - x)}\,dx. \quad \boxed{05}$$

$$I = \int_0^{\pi/2} \frac{\cos^2 x}{1+\cos^2 x}\,dx \quad \boxed{05}$$

$$\therefore \quad 0 = \int_0^{\pi/2} \frac{\sin^2 x}{1+\sin^2 x} - \frac{\cos^2 x}{1+\cos^2 x}\,dx \quad \boxed{05}$$

$$= \int_0^{\pi/2} \frac{\sin^2 x(1+\cos^2 x) - \cos^2 x(1+\sin^2 x)}{1+1+\sin^2 x\cos^2 x}\,dx$$

$$= \int_0^{\pi/2} \frac{\sin^2 x - \cos^2 x}{2+\sin^2 x\cos^2 x}\,dx \quad \boxed{05}$$

$$\therefore \int_0^{\pi/2} \frac{\cos 2x}{2+\sin^2 x\cos^2 x}\,dx = 0 \quad \boxed{05} \qquad \boxed{25}$$

(16) $l \equiv x+y+5=0$

$f(x,y) = x+y+5$

$f(-6,0) = -1 < 0$ 05

$f(-2,4) = 7 > 0$ 05

$\therefore$ A හා B, $l=0$ ට දෙපස පිහිටයි. 05 15

වෘත්තය මත වෙනත් ඕනෑම $P \equiv (x,y)$ ගනිමු.

(AP අනු)(BP අනු) $= -1$ 05

$$\left(\frac{y}{x+6}\right)\left(\frac{y-4}{x+2}\right) = -1$$ 05

$$y(y-4)+(x+6)(x+2)=0$$

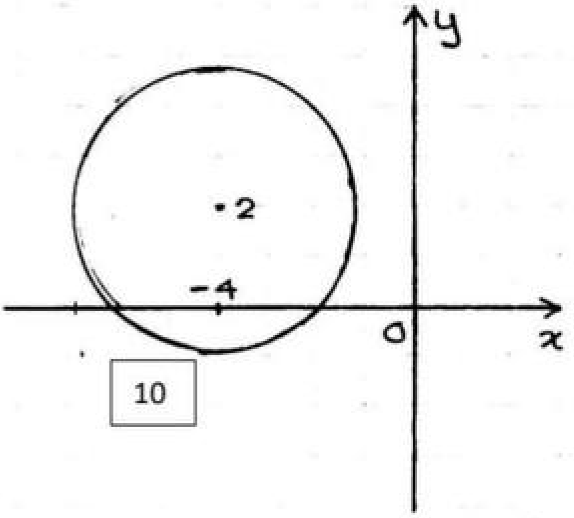
$$y^2-4y+x^2+8x+12=0$$

$$S_1 \equiv x^2+y^2+8x-4y+12=0$$ 05

කේන්ද්‍රය $C_1 \equiv (-4,2)$ 05

අරය $r = \sqrt{16+4-12}$

$= 2\sqrt{2}$ 05 25



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කේන්ද්‍රය C_1 සිට $l=0$ ට ලම්බ දුර d ගනිමු

$$d = \frac{|-4+2+5|}{\sqrt{1+1}}$$ 05

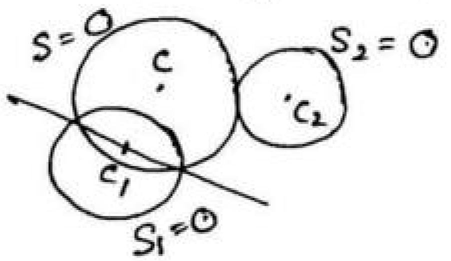
$$= \frac{\sqrt{3}}{2}$$ 05

$d < r$ 05

05

$\therefore S_1=0$ හා $l=0$ ඡේදනය වේ. 30

$S \equiv x^2+y^2+2gx+2fy+c=0$ ගනිමු.



$$S \equiv x^2+y^2+2gx+2fy+c=0$$
$$S_2 \equiv x^2+y^2-4=0$$

සමගාමීව විසඳීමෙන්

$$4+2gx+2fy+c=0$$

$$y = \frac{-(c+4)-2gx}{2f} \quad \boxed{10}$$

$$x^2+\frac{(c+4)^2+4g^2x^2+2(c+4)2gx}{4f^2}=4 \quad \boxed{05}$$

$$4f^2x^2+(c+4)^2+4g^2x^2+2\cdot 2g(c+4)x=16f^2$$

$$x^2(4g^2+4f^2)+4g(c+4)x+(c+4)^2-16f^2=0 \quad \boxed{05}$$

ස්පර්ශ වන නිසා $\Delta_x=0$ $\boxed{05}$

$$16g^2(c+4)^2-4(4g^2+4f^2)\left[(c+4)^2-16f^2\right]=0$$

$$g^2(c+4)^2-(g^2+f^2)\left[(c+4)^2-16f^2\right]=0$$

$$f^2\left[16g^2-(c+4)^2\right]=0 \quad \text{——} \quad ① \quad \boxed{10}$$

$S_1=0$ හා $S=0$ හි පොදු ජ්‍යාය

$$S-S_1=0 \quad \boxed{05}$$

$$(2g-8)x+(2f+4)y+c-12=0 \quad \boxed{05}$$

$(-4,2) \Rightarrow$

$$(2g-8)(-4)+(2f+4)2+c-12=0 \quad \boxed{05}$$

$$(-8g+32)+4f+8+c-12=0$$

$$c=8g-4f-28 \quad \boxed{05}$$

① ට ආදේශ

$$f^2\left[16g^2-(8g-4f-24)^2\right]=0 \quad \boxed{05}$$

$\boxed{05}$ $f=0$ හෝ $g^2-(2g-f-6)^2=0$ $\boxed{05}$

$$g^2-\left[4g^2-4g(f+6)+(f+6)^2\right]=0$$

$$-3g^2+4gf+24g+f^2-12f-36=0$$

$$3g^2+f^2-4gf+12f+36-24x=0$$

$$(g,f) \longrightarrow (x,y)$$

$$3x^2+y^2-4xy+24x+12y+36=0 \quad \boxed{10}$$

$\boxed{80}$

⑰ a) $\sin(A+B) = \sin A \cos B + \cos A \sin B$ $\boxed{05}$

$\cos(A+B) = \cos A \cos B - \sin A \sin B$ $\boxed{05}$

$$\begin{aligned}\sin 2x &= \sin(x+x) \quad \boxed{05}\\ &= \sin x \cos x + \sin x \cos x\\ &= 2\sin x \cos x \quad \boxed{05}\end{aligned}$$

$$\begin{aligned}\cos 2x &= \cos(x+x) \quad \boxed{05}\\ &= \cos x \cos x - \sin x \sin x\\ &= \cos^2 x - \sin^2 x \quad \boxed{05}\end{aligned}$$

$\boxed{30}$

$$\begin{aligned}f(x) &= \sqrt{3}\cos^2 x + 2\sin x\cos x - \sqrt{3}\sin^2 x\\ &= \sqrt{3}(\cos^2 x - \sin^2 x) + 2\sin x\cos x \quad \boxed{05}\\ &= \sqrt{3}\cos 2x + \sin 2x\\ &= 2\left(\frac{\sqrt{3}}{2}\cos 2x + \frac{1}{2}\sin 2x\right) \quad \boxed{05}\\ &= 2\left(\cos 2x \cos\frac{\pi}{6} + \sin 2x \sin\frac{\pi}{6}\right) \quad \boxed{05}\\ &= 2\cos\left(2x - \frac{\pi}{6}\right)\end{aligned}$$

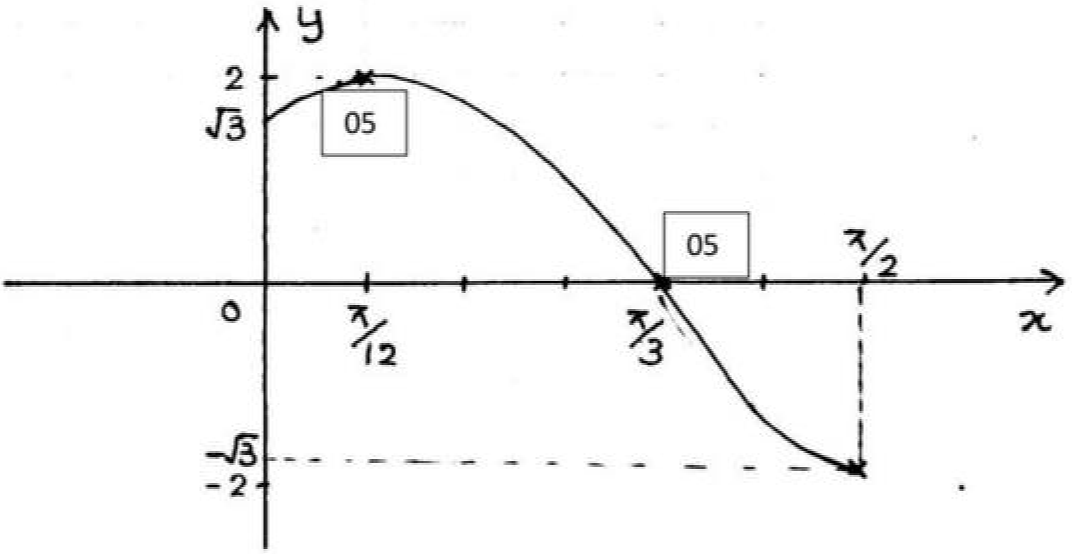
$\boxed{15}$

$$-1 \leqslant \cos(2x - \pi/6) \leqslant 1$$

$$-2 \leqslant 2\cos(2x - \pi/6) \leqslant 2 \quad \boxed{05}$$

$$f(0) = 2\cos(-\pi/6) = 2.\sqrt{3}/2 = \sqrt{3} \quad \boxed{05}$$

$$f(\pi/2) = 2\cos(\pi - \pi/6) = -2\cos\pi/6) = -\sqrt{3} \quad \boxed{05}$$

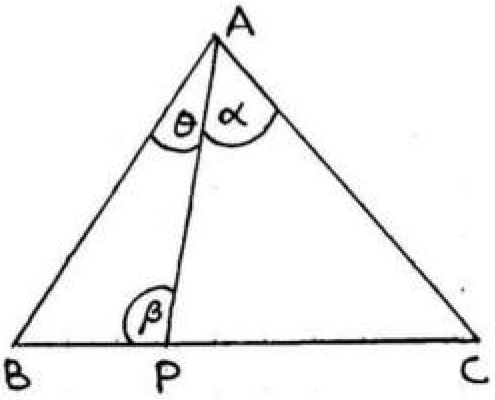


$\boxed{25}$

b) ඕනෑම ABC ත්‍රිකෝණයක් සඳහා සුපුරුදු අංකනයෙන්

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$ වේ. 05

05



ABP Δ න්

$$\frac{BP}{\sin\theta} = \frac{AB}{\sin\beta} = \frac{AP}{\sin B}$$ 05

$$BP = \frac{c\sin\theta}{\sin\beta}. \quad \text{——} ①$$ 05

10

APC Δ න්

$$\frac{PC}{\sin\alpha} = \frac{AC}{\sin(\pi-\beta)} = \frac{AP}{\sin C}$$ 05

$$PC = \frac{b\sin\alpha}{\sin\beta} \quad \text{——} ②$$ 05

①/② $$\frac{BP}{PC} = \frac{c\sin\theta}{b\sin\alpha}$$ 05

$$\frac{\sin A}{a} = \frac{\sin B}{b}$$

$$\frac{\sin A}{a} = \frac{AP\sin\theta}{BP} \times \frac{1}{b}.$$ 05

$$\frac{\sin A}{AP} = \frac{a\sin\theta}{BP.b}.$$ 05

$$= \left(\frac{BP+PC}{BP}\right)\frac{\sin\theta}{b}.$$

$$= \left(1 + \frac{b\sin\alpha}{c\sin\theta}\right)\frac{\sin\theta}{b}.$$ 05

$$= \frac{\sin\theta}{b} + \frac{\sin\alpha}{c}$$

30

(25)

c) $$\underbrace{2\tan^{-1}x}_{\alpha} + \underbrace{\tan^{-1}\left(\frac{x+1}{x+2}\right)}_{\beta .} = \pi/2$$

$$\tan\alpha = x \quad , \quad \tan\beta = \frac{x+1}{x+2} \quad \boxed{05}$$

$$2\alpha + \beta = \pi/2$$

$$2\alpha = \pi/2 - \beta .$$

$$\tan 2\alpha = \cot\beta . \quad \boxed{05}$$

$$\frac{2\tan\alpha}{1-\tan^2\alpha} = \frac{1}{\tan\beta}$$

$$\frac{2x}{1-x^2} = \frac{x+2}{x+1} \quad \boxed{05}$$

$$\frac{2x}{(1-x)(1+x)} = \frac{x+2}{(x+1)}$$

$$(x+1)\,2x = (x+1)\left[(x+2)(1-x)\right]$$

$$2x(x+1) = (x+1)(-x^2-x+2)$$

$$(x+1)(2x) - (x+1)(-x^2-x+2) = 0$$

$$(x+1)\left[2x+x^2+x-2\right] = 0 \quad \boxed{10}$$

$$(x+1)(x^2+3x-2) = 0$$

$x = -1$ හෝ $x = \dfrac{-3 \pm \sqrt{9+8}}{2}$ $\boxed{05}$

$x = -1$ හෝ $x = \dfrac{-3 \pm \sqrt{17}}{2}$

$$x \neq -1 \ , \ \frac{-3-\sqrt{17}}{2}$$

$$\therefore x = \frac{-3+\sqrt{17}}{2.} \quad \boxed{05}$$

$$\boxed{35}$$