



No :

Date :

ആംഗിതം ത്രികോണമിതി = $(\theta + \theta)$ നിമി

ആംഗിതം - θ ആംഗിതം = $(\theta - \theta)$ നിമി

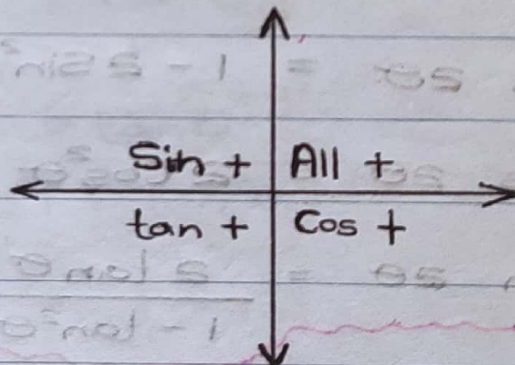
θ	0°	90°	180°	270°
$\sin \theta$	0	1	0	-1
$\cos \theta$	1	0	-1	0
$\tan \theta$	0	അവ്യക്തം	0	അവ്യക്തം

$\pi/2$ ↑
 $\sin = \cos$
 $\cos = \sin$
 $\tan = \cot$

↓ $3\pi/2$
 $\sin = \cos$
 $\cos = \sin$
 $\tan = \cot$

$\sin = \sin$
 $\cos = \cos$
 $\tan = \tan$

$\sin = \sin$
 $\cos = \cos$
 $\tan = \tan$



$$\sin^2 \theta + \cos^2 \theta = 1$$

$$1 + \tan^2 \theta = \sec^2 \theta$$

$$1 + \cot^2 \theta = \operatorname{cosec}^2 \theta$$



No :

Date :

$$\sin(A+B) = \sin A \cos B + \sin B \cos A$$

$$\sin(A-B) = \sin A \cos B - \sin B \cos A$$

$$\cos(A+B) = \cos A \cos B - \sin A \sin B$$

$$\cos(A-B) = \cos A \cos B + \sin A \sin B$$

$$\tan(A+B) = \frac{\tan A + \tan B}{1 - \tan A \tan B}$$

$$\tan(A-B) = \frac{\tan A - \tan B}{1 + \tan A \tan B}$$

$$\sin 2\theta = 2 \sin \theta \cos \theta$$

$$\cos 2\theta = \cos^2 \theta - \sin^2 \theta$$

$$\cos 2\theta = 1 - 2 \sin^2 \theta$$

$$\cos 2\theta = 2 \cos^2 \theta - 1$$

$$\tan 2\theta = \frac{2 \tan \theta}{1 - \tan^2 \theta}$$

$$\sin 3\theta = 3 \sin \theta - 4 \sin^3 \theta$$

$$\cos 3\theta = 4 \cos^3 \theta - 3 \cos \theta$$

$$\tan 3\theta = \frac{3 \tan \theta - \tan^3 \theta}{1 - 3 \tan^2 \theta}$$



No :

Date :

$$\sin A + \sin B = 2 \sin \left(\frac{A+B}{2} \right) \cos \left(\frac{A-B}{2} \right)$$

$$\sin A - \sin B = 2 \cos \left(\frac{A+B}{2} \right) \sin \left(\frac{A-B}{2} \right)$$

$$\cos A + \cos B = 2 \cos \left(\frac{A+B}{2} \right) \cos \left(\frac{A-B}{2} \right)$$

$$\cos A - \cos B = -2 \sin \left(\frac{A+B}{2} \right) \sin \left(\frac{A-B}{2} \right)$$

$$2 \sin A \cos B = \sin(A+B) + \sin(A-B)$$

$$2 \cos A \sin B = \sin(A+B) - \sin(A-B)$$

$$2 \cos A \cos B = \cos(A+B) + \cos(A-B)$$

$$2 \sin A \sin B = \cos(A-B) - \cos(A+B)$$

$$\cos 2\theta = \frac{1 - \tan^2 \theta}{1 + \tan^2 \theta}$$

$$\sin 2\theta = \frac{2 \tan \theta}{1 + \tan^2 \theta}$$



No :

Date :

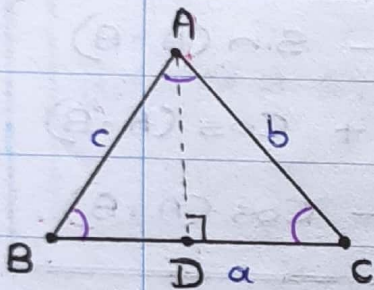
සයවිති නීතිය

ABC ත්‍රිකෝණයේ ත්‍රිකෝණයක උපරිම දිගකින්,

$$\frac{\sin A}{a} = \frac{\sin B}{b} = \frac{\sin C}{c}$$

පාදකය ...

පාදකයේ Δ ඇත



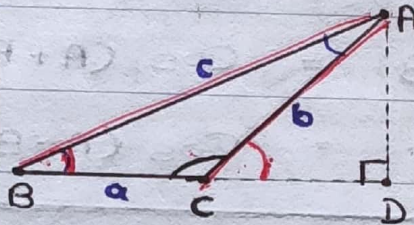
ABD Δ ගේ,

$$AD = c \sin B \text{ --- ①}$$

ADC Δ ගේ,

$$AD = b \sin C \text{ --- ②}$$

පාදකයේ Δ ඇත



ABD Δ ගේ,

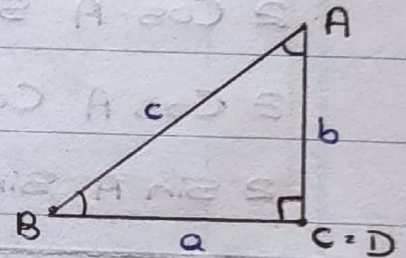
$$AD = c \sin B \text{ --- ①}$$

ACD Δ ගේ,

$$\sin(\pi - C) = \frac{AD}{b}$$

$$AD = b \sin C \text{ --- ②}$$

පාදකයේ Δ ඇත



ABD Δ ගේ

$$AD = c \sin B \text{ --- ①}$$

$$AD = b$$

$$AD = b \sin \pi/2$$

$$AD = b \sin C \text{ --- ②}$$

① හා ②,

$$c \sin B = b \sin C$$

$$\frac{\sin B}{b} = \frac{\sin C}{c}$$

එසේම, $\frac{\sin C}{c} = \frac{\sin A}{a}$ බව පෙන්විය හැකිය.

$$\frac{\sin A}{a} = \frac{\sin B}{b} = \frac{\sin C}{c}$$



No:

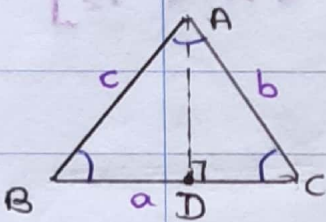
Date:

කෝසයින් ගිණියලියවිල්ලේ රෙගුලය

ප්‍රතිකූල ත්‍රිකෝණයක් ගත්කොට $ABC \Delta$ යන,

$$* \cos A = \frac{b^2 + c^2 - a^2}{2bc} \quad * \cos B = \frac{a^2 + c^2 - b^2}{2ac}$$

$$* \cos C = \frac{a^2 + b^2 - c^2}{2ab}$$

ප්‍රාග්ධනයප්‍රථම කෝණී Δ සඳහා

$$DC = b \cos C$$

 $ABD \Delta$ යන,

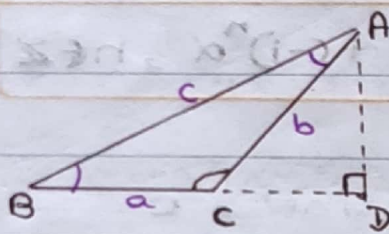
$$AB^2 = AD^2 + BD^2$$

$$c^2 = AC^2 - DC^2 + (BC - DC)^2$$

$$c^2 = b^2 - DC^2 + BC^2 - 2BC \cdot DC + DC^2$$

$$c^2 = b^2 + a^2 - 2a \cdot b \cdot \cos C$$

$$\cos C = \frac{b^2 + a^2 - c^2}{2ba}$$

මහා කෝණී Δ සඳහා

$$\cos(\pi - C) = \frac{CD}{b}$$

$$CD = -b \cos C$$

 $ABD \Delta$ නි

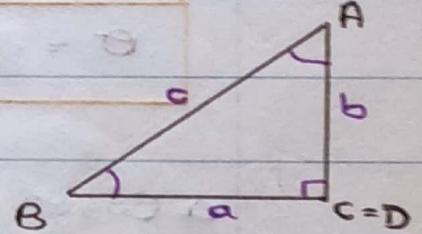
$$AB^2 = AD^2 + BD^2$$

$$c^2 = AC^2 - DC^2 + (BC + CD)^2$$

$$c^2 = b^2 + BC^2 + 2(BC)(CD)$$

$$c^2 = b^2 + a^2 + 2a(-b \cos C)$$

$$\cos C = \frac{a^2 + b^2 - c^2}{2ab}$$

සෘජුකෝණී Δ සඳහා $ABD \Delta$ නි

$$AB^2 = AD^2 + BD^2$$

$$c^2 = b^2 + a^2$$

$$c^2 = b^2 + a^2 - 2ab \cos C$$

$$c^2 = b^2 + a^2 - 2ab \cos C$$

$$\cos C = \frac{a^2 + b^2 - c^2}{2ab}$$

$$\text{එබැවින්, } \cos A = \frac{b^2 + c^2 - a^2}{2bc}$$

$$\cos B = \frac{a^2 + c^2 - b^2}{2ac}$$

බිමද සාධනය
සමාන වේ.



No :

Date :

සමකෝණීය ශ්‍රේණි

$$\tan \theta = \tan \alpha$$

$$\left[-\frac{\pi}{2} \leq \alpha \leq \frac{\pi}{2}\right]$$

$$\theta = n\pi + \alpha, n \in \mathbb{Z}$$

$$\cos \theta = \cos \alpha$$

$$[0 \leq \alpha \leq \pi]$$

$$\theta = 2n\pi \pm \alpha, n \in \mathbb{Z}$$

$$\sin \theta = \sin \alpha$$

$$\left[-\frac{\pi}{2} < \alpha < \frac{\pi}{2}\right]$$

$$\theta = n\pi + (-1)^n \alpha, n \in \mathbb{Z}$$

$$a \cos \theta + b \sin \theta = c$$

$$a \cos \theta + b \sin \theta = c$$

$\sqrt{a^2+b^2}$ නි ගුණකය,

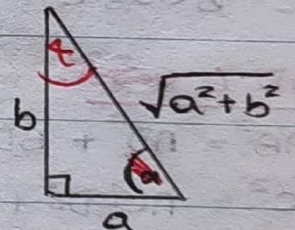
$$\frac{a}{\sqrt{a^2+b^2}} \cos \theta + \frac{b}{\sqrt{a^2+b^2}} \sin \theta = \frac{c}{\sqrt{a^2+b^2}}$$

$$\sin \alpha \cos \theta + \cos \alpha \sin \theta = \frac{c}{\sqrt{a^2+b^2}}$$

$$\sin(\alpha + \theta) = \sin \gamma$$

$$\alpha + \theta = n\pi + (-1)^n \gamma$$

$$\theta = n\pi + (-1)^n \gamma - \alpha, n \in \mathbb{Z}$$



$$\sin \alpha = \frac{a}{\sqrt{a^2+b^2}}$$

$$\cos \alpha = \frac{b}{\sqrt{a^2+b^2}}$$

$$\sin \gamma = \frac{c}{\sqrt{a^2+b^2}}$$



No :

Date :

ଉତ୍ତର ୦୧

* $y = \sin^4 x + \cos^4 x$ ର ପରିସର ନିର୍ଣ୍ଣୟ କର ।

$$y = (\sin^2 x + \cos^2 x) - 2\sin^2 x \cos^2 x = 1 - 2\sin^2 x \cos^2 x$$

$$y = 1 - 2\sin^2 x \cos^2 x$$

$$y = 1 - \frac{1}{2} \sin^2 2x$$

$$y = 1 - \frac{1}{2} \left(\frac{1 - \cos 4x}{2} \right)$$

$$y = 1 - \frac{1}{4} + \frac{1}{4} \cos 4x$$

$$y = \frac{1}{4} \cos 4x + \frac{3}{4}$$

$$-1 \leq \cos 4x \leq 1$$

$$-\frac{1}{4} \leq \frac{1}{4} \cos 4x \leq \frac{1}{4}$$

$$\frac{3}{4} - \frac{1}{4} \leq \frac{1}{4} \cos 4x + \frac{3}{4} \leq \frac{1}{4} + \frac{3}{4}$$

$$\frac{2}{4} \leq \frac{1}{4} \cos 4x + \frac{3}{4} \leq 1$$

ଉପର ସୀମା

ନିମ୍ନ ସୀମା

$$\cos 4x = 1$$

$$\cos 4x = -1$$

$$\cos 4x = \cos 0$$

$$\cos 4x = \cos \pi$$

$$4x = 2n\pi$$

$$4x = 2n\pi \pm \pi$$

$$x = \frac{2n\pi}{4}, n \in \mathbb{Z}$$

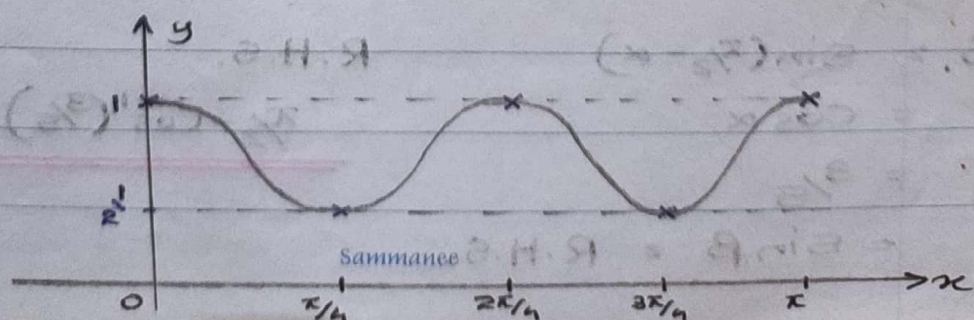
$$x = \frac{1}{4}(2n\pi \pm \pi)$$

$$n=0, x=0$$

$$n=0, x=\frac{\pi}{4} \text{ or } x=-\frac{\pi}{4}$$

$$n=1, x=\frac{\pi}{2}$$

$$n=1, x=\frac{3\pi}{4} \text{ or } x=\frac{5\pi}{4}$$





No :

Date :

ප්‍රතිලෝම ත්‍රිකෝණමිතික ශ්‍රේණි

* $y = \sin^{-1} x$ හි ශ්‍රේණි

$$-\pi/2 \leq \sin^{-1} x \leq \pi/2$$

ප්‍රතිලෝම ශ්‍රේණි.

* $y = \cos^{-1} x$ හි ශ්‍රේණි

$$0 \leq \cos^{-1} x \leq \pi$$

ප්‍රතිලෝම ශ්‍රේණි.

* $y = \tan^{-1} x$ හි ශ්‍රේණි

$$-\pi/2 < \tan^{-1} x < \pi/2$$

ප්‍රතිලෝම ශ්‍රේණි.

උදා:- 1) $\pi/2 - \cos^{-1}(3/5) = \sin^{-1}(3/5)$ බව පෙන්වන්න.

$$\cos^{-1}(3/5) = \alpha$$

$$\sin^{-1}(3/5) = \beta$$

$$\cos \alpha = 3/5$$

$$\sin \beta = 3/5$$

$$\pi/2 - \alpha = \beta$$

$$\sin(\pi/2 - \alpha) = \sin \beta$$

$$\text{L.H.S.} = \sin(\pi/2 - \alpha)$$

$$= \cos \alpha$$

$$= 3/5$$

$$= \sin \beta = \text{R.H.S.}$$

$$\pi/2 - \cos^{-1}(3/5) = \sin^{-1}(3/5)$$



No :

Date :

$$2) \tan^{-1} x = 2 \tan^{-1} [\operatorname{cosec}(\tan^{-1} x) - \tan(\cot^{-1} x)] \quad \text{or}$$

$$\tan^{-1} x = \alpha \quad \cot^{-1} x = \beta$$

$$\tan \alpha = x \quad \cot \beta = x$$

$$\alpha = 2 \tan^{-1} (\operatorname{cosec} \alpha - \tan \beta)$$

$$\alpha/2 = \tan^{-1} (\operatorname{cosec} \alpha - \tan \beta)$$

$$\tan \alpha/2 = \operatorname{cosec} \alpha - \tan \beta \quad \text{or}$$

$$\text{R.H.S.} = \operatorname{cosec} \alpha - \tan \beta$$

$$(\tan \alpha = x = \cot \beta)$$

$$= \operatorname{cosec} \alpha - \frac{1}{\tan \alpha}$$

$$= \frac{1}{\sin \alpha} - \frac{\cos \alpha}{\sin \alpha} = \frac{1 - \cos \alpha}{\sin \alpha}$$

$$= \frac{1 - (1 - 2 \sin^2 \alpha/2)}{2 \sin \alpha/2 \cos \alpha/2} = \frac{2 \sin^2 \alpha/2}{2 \sin \alpha/2 \cos \alpha/2}$$

$$= \tan \alpha/2 = \text{L.H.S.}$$



No :

Date :

* $c^2 \leq a^2 + b^2$ කළ, a, b හා c තාත්වික නිශාන්තයන් වන විට $a \cos x + b \sin x = c$ සමීකරණයේ x සඳහා තාත්වික විසඳුම් නැති බව ප්‍රවේශනය කළ හැක. (මෙය $a \cos x + b \sin x = \sqrt{a^2 + b^2} \sin(x + \alpha)$ වේ.)

$$(a \cos x + b \sin x) = c$$

$$(a \cos x + b \sin x) = \sqrt{a^2 + b^2} \sin(x + \alpha) = c$$

$$\sin(x + \alpha) = \frac{c}{\sqrt{a^2 + b^2}}$$

තාත්වික වීමට - $-1 \leq \sin(x + \alpha) \leq 1$

$$-1 \leq \sin(x + \alpha) \leq 1$$

$$|\sin(x + \alpha)| \leq 1$$

$$\left| \frac{c}{\sqrt{a^2 + b^2}} \right| \leq 1$$

$$c \leq \sqrt{a^2 + b^2}$$

$$c^2 \leq a^2 + b^2$$

Sin ඡාත්‍ය නිධනය හර Cos ඡාත්‍ය ප්‍රාථමිකය

* පළමුව Sin ඡාත්‍ය ලියා ගත යුතුය.

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C} = \lambda \text{ ලෙස ගනිමු,}$$

එනම්

$$\frac{\sin A}{a} = \frac{\sin B}{b} = \frac{\sin C}{c} = \mu \text{ ලෙස ගනිමු.}$$

මෙම ඉකුර 2 හි පහළ ඉකුරයක් ගත යුතුය.

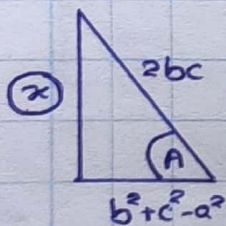
* තවුරු පෙනවිය යුතුවේ $a^2 = b^2 + c^2 - 2bc \cos A$ නිසා,
මෙය පහළ ලෙස සකස් කිරීම.

$$\underbrace{2bc \cos A}_{\text{L.H.S.}} = \underbrace{b^2 + c^2 - a^2}_{\text{R.H.S.}}$$

මෙය R.H.S. ගෙන් පහළ ගෙන Sin ඡාත්‍යයෙන්
ප්‍රාථමිකය ලෙස L.H.S. ලියා ගත යුතුය.

Cos ඡාත්‍යයෙන් Sin ඡාත්‍ය ප්‍රාථමිකය

$$\cos A = \frac{b^2 + c^2 - a^2}{2bc}$$

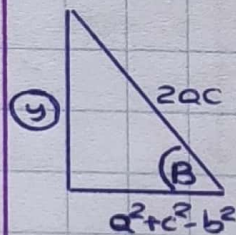


පරිමාණය
ලෙස (x)
සකස් කිරීම.

$$\sin A = \frac{x}{2bc}$$

$$\frac{\sin A}{a} = \frac{x}{2bca}$$

$$\cos B = \frac{a^2 + c^2 - b^2}{2ac}$$



පරිමාණය
(y) සකස් කිරීම.

$$\sin B = \frac{y}{2ac}$$

$$\frac{\sin B}{b} = \frac{y}{2abc}$$

මෙයින් $x = y$ ලෙස විමසීම ලැබේ.

$$\therefore \frac{\sin A}{a} = \frac{\sin B}{b} \text{ ලෙස ලිවිය හැක.}$$

Atlas